

Barnes

Arithmetic We Need

[illegible]

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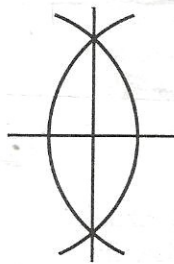
GUY T. BUSWELL

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Arithmetic We Need

Enlarged Edition



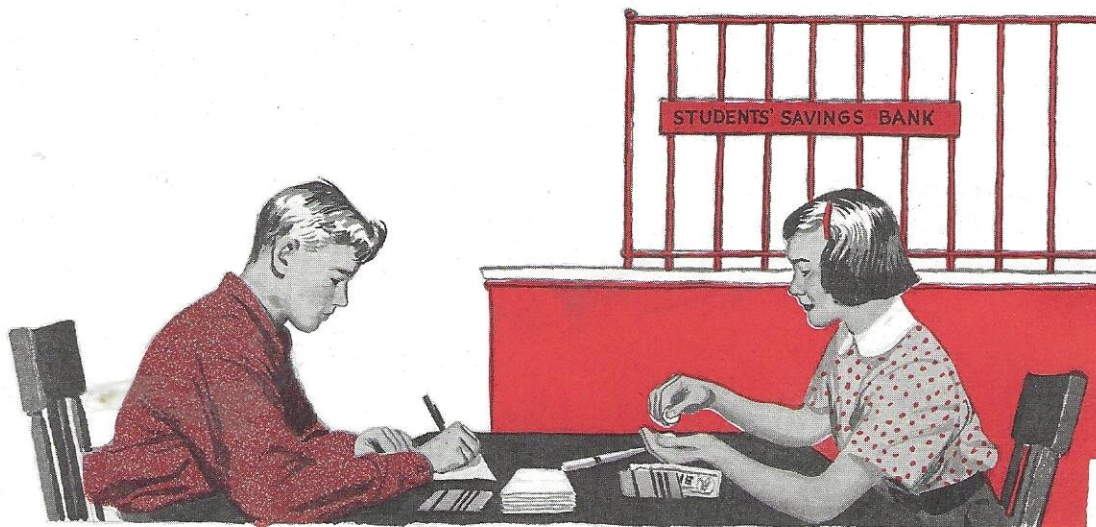
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The Excitement of Atomic Power!

Inventory [W]

You may hear people talk a great deal about the atom, atomic power, and the atomic bomb. Jack and Doug like to think out what they might be doing when atomic energy is used to make fuel for rocket ships like the one which is approaching the space station in the picture. The boys worked out answers to these questions. Can you?

1. If regular space-ship interplanetary travel becomes possible in the year 2050, in how many years will that be?

2. Many people think that an exploratory trip around the moon and back again might be made earlier, in about the year 2000. How many years from now will that be?

3. The nearest point on one of the planets to which Doug thought it would be fun to travel is 26,000,000 miles away. Write this number in words.

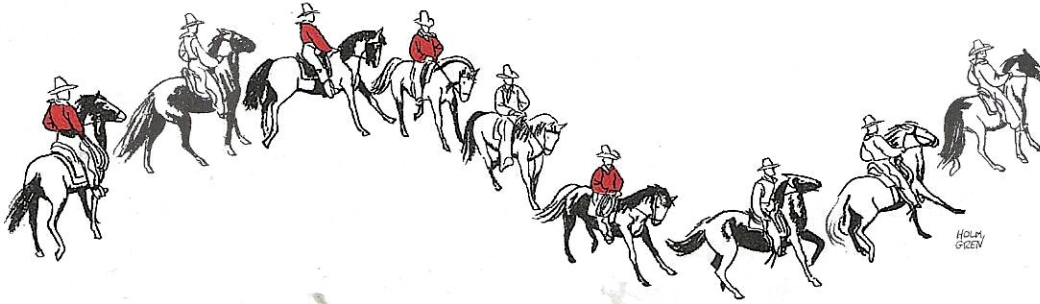
4. Jack read that such a trip some day might require about 1,000 hours. About how many 24-hour days would this be?

5. About how many miles an hour would a rocket space ship have to average to cover 26,000,000 miles in 1,000 hours?

6. Scientists have figured that the cost of building a satellite space station about 1,000 miles above the earth might be \$4,000,000,000. Write this number in words. Write it also with some figures and the words, "millions of dollars."

7. The speed at which the space station will need to travel in its revolutions around the earth is a little more than 4 miles a second. About how many miles an hour will the space station move?

8. Scientists have figured that in order to escape the gravitational pull of the earth, a space ship should start from earth toward the space station at the rate of about 7 miles per second. This would be at the rate of about how many miles an hour?



Cardinal and Ordinal Numbers

[O] Very great distances and speeds, such as you were thinking about on page 1, require an accurate use of numbers.

Before we get into the year's work, let's make sure about the correct use of numbers and the terms used with them.

1. **How many** horses are stepping along in line?

Numbers telling how many are cardinal numbers.

2. Counting from the right, riders with red jackets are **in what positions** in the line?

We count to find numbers telling positions. However, instead of saying that riders with red jackets are in 4, 6, 7, and 9 positions, we say that they are in fourth (4th), sixth (6th), seventh (7th), and ninth (9th) positions. Does "9th" tell how many or does it tell where something comes in a series?

Numbers telling order or position are ordinal numbers.

3. Suppose you live at 315 Maple Street. Is 315 an ordinal number or a cardinal number? Why?

4. In "page 60" is 60 a cardinal or an ordinal number?

5. From the following, make lists of the cardinal numbers and the ordinal numbers: a. 12 oranges; b. the 4th house from the corner; c. 30 days; d. September 29; e. the seventh day; f. 175 bushels of corn; g. 18¢; h. the 20th question; i. 26,000,000 miles; j. the year 2500.

Prime and Composite Numbers

[O]

You know that all whole numbers are either **even** numbers or **odd** numbers. Odd numbers cannot be divided exactly by 2.

1. Name four even numbers; five odd numbers.

2. Which of the following are odd and which are even numbers? 12, 17, 250, 478, 15, 999, 500, 1,001

All whole numbers may be classified as either **prime** numbers or **composite** numbers. A prime number cannot be divided evenly by any number except itself and 1.

3. Give three prime numbers.

A composite number is one that can be divided evenly by a number other than 1 or itself. 12 is a composite number because it can be divided by any one of its factors, 2 and 6; 3 and 4; 2 and 2 and 3.

4. For 12, which set of factors contains all prime numbers? Why are these called the **prime factors** of 12?

5. Give two composite numbers. Tell their prime factors.

6. Is 9 a prime number? Why?

7. Are 4 and 7 prime factors of 28? Why or why not?

8. Are 2, 2, and 7 prime factors of 28? Explain.

When composite numbers are broken up into their prime factors, the process is called **factoring**.

[W]

Copy from the following the numbers which have not been broken up into prime factors. Write their prime factors.

9. $10 = 2 \times 5$

13. $30 = 2 \times 15$

10. $12 = 3 \times 4$

14. $25 = 5 \times 5$

11. $30 = 3 \times 2 \times 5$

15. $18 = 3 \times 6$

12. $100 = 10 \times 10$

16. $75 = 3 \times 5 \times 5$

17. Which of the following numbers are composite and which are prime? 19, 24, 32, 29, 52, 13, 27, 37

A				
Place Value	Hundred billion Ten billion Billion	Hundred million Ten million Million	Hundred thousand Ten thousand Thousand	Hundred Ten One
Number	7 0 5 ,	3 2 1 ,	5 4 6 ,	8 7 9
Period Value	Billion	Million	Thousand	One

Reading and Writing Whole Numbers

Showing meaning [O]

1. See if you can read the number in box A.
2. In box B, read the value of each figure of the number.

B

705,321,546,879 is the sum of

7 hundred billions,	or	700,000,000,000
and 0 ten billions,	or	00,000,000,000
and 5 billions,	or	5,000,000,000
and 3 hundred millions,	or	300,000,000
and 2 ten millions,	or	20,000,000
and 1 million,	or	1,000,000
and 5 hundred thousands,	or	500,000
and 4 ten thousands,	or	40,000
and 6 thousands,	or	6,000
and 8 hundreds,	or	800
and 7 tens,	or	70
and 9 ones,	or	9

705,321,546,879

3. As in box B, write the meaning of: 95,274; 825,476. [W]

The period at the left of billions is **trillion's period**.

4. See if you can write the number for 52 trillions.

Here are other ways to show the meaning of a number.

C

47,206 means

4 ten thousands, 7 thousands, 2 hundreds, 0 tens and 6 ones

or 47 thousands, 2 hundreds, 0 tens and 6 ones

or 472 hundreds, 0 tens and 6 ones

or 4,720 tens and 6 ones

or 47,206 ones

Write the following in figures. When there is no amount to write in a given place in a number, we use 0 in that place.

5. 3 hundreds, 7 tens, 6 ones
6. 8 hundreds, 0 tens, 2 ones
7. 1 thousand, 7 hundreds, 4 ones
8. 475 hundreds, 5 ones
9. 10 hundreds, 8 ones
10. 6 thousands, seven ones
11. 75 thousands, 4 hundreds, 0 tens, 1 one
12. 18 tens, 4 ones
13. 1 thousand, 1 one

What Do You Know about Zero?

Two uses [O]

1. In 204, we say that zero is a **place-holder**. Explain.
2. Zero may also be a **number** that tells *how many*. If a baseball score is 3 to 0, explain how both the 3 and the 0 tell how many.

Tell whether zero is place-holder or number in Ex. 3–6.

3. A number has 3 in one's place and 6 in thousand's place, and the other figures are zeros.
4. $6 + 0 + 9 + 4 = ?$
5. $\$3.75 - 0 = ?$
6. The first figure of a number is 2 and the last figure is 2 and there are four 0's between the 2's.

Addition and Multiplication

Meaning; relationship; terms; order; check [O]

1. What process is used to find the total of *unequal groups* of *like-things*?

2. We can find a total by counting. Why is adding faster?

3. What do we call the numbers that we add? the total?

4. $7 + 9 + 4 = 20$; so $9 + 7 + 4 = ?$ What do you know about a sum when the *order of the addends* is changed?

A

61	}	addends
61		
61		
61		
61		
+61		
305		sum

B

61	<i>multiplicand</i>
×5	<i>multiplier</i>
305	<i>product</i>

5. How can you check an example in addition?

6. To find how much five 61's are, why could you either add as in box A or multiply as in box B?

7. In box B, the multiplicand shows the size of the *equal groups*. What does the multiplier show? What does the product show?

8. $3 \times 5 \times 2 = 30$, so $5 \times 3 \times 2 = ?$ What happens to the product when we change the *order of factors*? Why is multiplying after changing the order of factors a good check?

➤ **Extra Practice.** Turn to pages 299 and 302. Cover the answers and quickly say all the facts in Sets 1 and 11.

When you know all the addition facts, you can quickly add any 2-place number to a 1-place number.

Give sums for these as rapidly as you can:

	a	b	c	d	e	f	g
9.	16	15	24	86	8	76	39
	+2	+4	+7	+5	+42	+4	+5

➤ **Extra Practice.** Say the sums in Set 2.

10. Box D shows what we do when we **carry in the addition** example in box C. Explain the addition and the carrying.

C	D
Hundreds Tens Ones 4 5 6 3 8 9 + 7 6 8 ----- 1,6 1 3	Hundreds Tens Ones 4 hundreds and 5 tens and 6 ones 3 hundreds and 8 tens and 9 ones + 7 hundreds and 6 tens and 8 ones ----- 16 hundreds and 1 tens and 3 ones

11. Box E shows how we carry in multiplication. Explain.

E	F	G	H
Tens Ones 2 6 × 3 ----- 6 8 ----- 7 8	1,8 4 2 × 4 ----- 7,3 6 8	82 × 37 ----- 574 246 ----- ?	472 × 203 ----- 1 416 94 40 ----- 95,816

12. In box F, explain the multiplying and the carrying.

13. In box G, the second partial product means 246 tens, or 2,460. Explain.

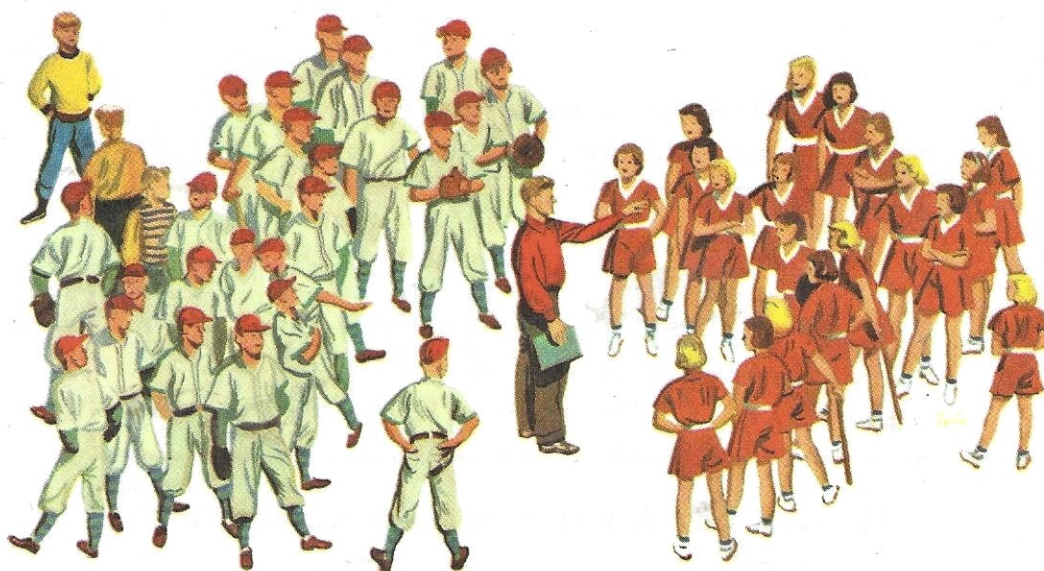
14. In box H, how do we show the multiplying by zero?

[W]

Copy, work, and check.

15. 9 16. 8 17. 3 18. 147 19. 236 20. 603
 1 7 8 670 ×2 ×29
 5 9 5 342
 4 3 0
 0 9 2
 4 6 9 21. 1,795 22. 9,208 23. 2,005
 3 5 9 ×40 ×375 ×4,937

➤ **Extra Practice.** Work Sets 3–4 and 12–15.



At the Playground

Differentiating A. and M. [O]

To find the total, we add when the groups are unequal in size and multiply when the groups are equal.

Tell for each problem why you would add or multiply.

1. From the playground groups, Mr. Spencer chose 24 boys for the school baseball squad and 19 girls for the softball squad. How many were chosen in all?
2. The boys' uniforms cost \$7.50 each. How much would uniforms cost for all the boys who played baseball?
3. The girls' uniforms cost \$2.75 each. How much would all the uniforms for the softball players cost?
4. What was the total amount needed for uniforms?
5. In order to pay for the uniforms, a committee planned to have each of the 204 students on the playground sell 5 tickets for the first game at 35¢ a ticket. If all these tickets were sold, how much would be received for uniforms?

[W]

Now solve problems 1 to 5 above.

Subtraction and Division

Meaning; terms [O]

1. When we know how many there are in a total group and how many there are in one of its two parts, what do we do to find how many there are in the other part?

2. What do we call the number from which we subtract? the number subtracted? the answer?

A

$$\begin{array}{r} 5,704 \text{ minuend} \\ -2,356 \text{ subtrahend} \\ \hline 3,348 \text{ remainder} \\ \text{or difference} \end{array}$$

3. In box A, see if you can explain what each name means.

5,704 is the minuend because . . .

4. When we must take away several equal groups (or numbers), what process should we use?

Division helps us to answer these two kinds of questions:

a. **Measurement division.** How many equal parts of a certain size are there in a group? (How many 4's in 20?)

b. **Fractional-part division.** What is the size of each part when a group is divided into equal parts? ($\frac{1}{4}$ of 20 = ?)

5. What is the answer in division called? What is the number you divide by called? the number you divide?

6. Which number is the divisor in box B? in $84 \div 12$? in $\frac{90}{3}$?

B

$$\begin{array}{r} 352 \text{ quotient} \\ \text{divisor } 7 \overline{)2,464} \text{ dividend} \end{array}$$

7. In box B, the dividend shows thousands. Why is there no thousand's figure in the quotient?

8. Study the short-division work in box B. The first division, 24 (hundreds) $\div$ 7, is uneven. What table number helps you to get the quotient figure 3 (hundreds)?

9. After the first division in B, how many hundreds are left and combined with 6 tens? After the second division, how many tens are left and combined with 4 ones?

Review of Subtraction

Reasons for subtracting; facts [O]

$$\begin{array}{r} 369 \text{ minuend} \\ -235 \text{ subtrahend} \\ \hline 134 \text{ remainder or} \\ \text{difference} \end{array}$$

Look at the subtraction example in the box. You know that the minuend, 369, is the sum of 235 and another number. **You subtract to find the other number.**

Subtraction is often used to answer the questions "How many are gone?" and "How many are left?" The answer to either of these questions is called the **remainder**.

Many times also we subtract to compare two numbers. That is, we answer questions such as "How many more (or less) is one number than another number?" or "How much more is needed?" The answer when we compare two numbers by subtraction is called the **difference**.

Tell why you subtract in each of the following problems. Is the answer a remainder or a difference?

1. Sally gave Joyce 16 of her 48 Canadian stamps. How many Canadian stamps does Sally have left?
2. One rainy day Ted sorted his books and picked out some to give to the children's room in the library. When he began he had 87 books and when he finished he had 72. How many books did he give to the library?
3. Joe has saved \$23.50 to buy a bicycle. The bicycle costs \$38.95. How much more must Joe save?
4. Mt. McKinley in Alaska is 20,269 ft. high and Mt. Logan in Canada is 19,850 ft. high. How much higher is Mt. McKinley than Mt. Logan?

➤ **Extra Practice.** Test yourself by covering the answers and saying the subtraction facts in Set 5.

If you know all the subtraction facts, you can quickly subtract any 1-place number from a 2-place number. Turn to Set 6 and say the remainders.

Borrowing; check [O]

Besides knowing the facts, you must know how to borrow.

When you carry in addition, you change 10 ones to 1 ten or 10 tens to 1 hundred, and so on. When you borrow, you do just the opposite—you change 1 of the tens in the minuend to 10 ones or 1 of the hundreds to 10 tens, and so on.

A	B	C	D
$\begin{array}{r} \text{Tens} \quad \text{Ones} \\ 3 \quad 15 \\ \cancel{4} \cancel{5} \\ - 28 \\ \hline 17 \end{array}$	$\begin{array}{r} \text{Hundreds} \quad \text{Tens} \quad \text{Ones} \\ 6 \quad 13 \\ \cancel{7} \cancel{8} 6 \\ - 293 \\ \hline 443 \end{array}$	$\begin{array}{r} \text{Thousands} \quad \text{Hundreds} \quad \text{Tens} \quad \text{Ones} \\ 3 \quad 12 \quad 12 \\ \cancel{4} \cancel{8} \cancel{2} 9 \\ - 2,684 \\ \hline 1,645 \end{array}$	$\begin{array}{r} 6 \quad 9 \quad 14 \\ \cancel{7} \cancel{0} \cancel{4} \\ - 378 \\ \hline 326 \end{array}$

5. In box A, 1 ten is borrowed from $__$ tens, leaving $__$ tens. The 1 ten is changed to 10 ones and added to the 5 ones. Are 3 tens and 15 ones equal to 4 tens and 5 ones?

6. In box B, do you need to borrow? Explain the work.

7. In box C, you must borrow twice. Explain the work.

8. In box D, the minuend has how many tens in all? We cannot borrow from 0 tens, so we think, "There are 70 tens. There will be $__$ tens left when we have borrowed 1 ten."

To check Ex. A, we add 17 and 28. Why should the sum of these equal 45? Explain how to check Ex. B–D.

[W]

Subtract and check. In Ex. 9–13 show your borrowing. Work Ex. 14–18 without showing the borrowing.

$$\begin{array}{r} 9. \quad 34 \\ - 27 \\ \hline \end{array} \quad \begin{array}{r} 10. \quad 87 \\ - 38 \\ \hline \end{array} \quad \begin{array}{r} 11. \quad 305 \\ - 156 \\ \hline \end{array} \quad \begin{array}{r} 12. \quad 619 \\ - 247 \\ \hline \end{array} \quad \begin{array}{r} 13. \quad 8,327 \\ - 1,937 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 125 \\ - 67 \\ \hline \end{array} \quad \begin{array}{r} 15. \quad 600 \\ - 587 \\ \hline \end{array} \quad \begin{array}{r} 16. \quad 5,312 \\ - 903 \\ \hline \end{array} \quad \begin{array}{r} 17. \quad 7,153 \\ - 4,638 \\ \hline \end{array} \quad \begin{array}{r} 18. \quad 8,258 \\ - 3,792 \\ \hline \end{array}$$

► **Extra Practice.** Work Sets 7–10.

Review of Division

Division facts [O]

Turn to Extra Practice Set 16, cover the quotients, and say the division facts as quickly as you can. Write down and study any combinations that give you trouble.

Meaning of division

On this page, you divide to find (a) how many times one group is contained in another (**measurement division**) or (b) how many there are in each equal part of a group (**fractional-part division**).

Which kind of division (measurement or fractional-part) is each of the following?

1. Each day Mr. Spencer used $\frac{1}{4}$ of his 32 students for playground duty. How many students were in each squad?
2. Mr. Spencer asked Janet to distribute 5 sheets of paper to each student. To how many could Janet give 5 sheets from a pile of 150 sheets?
3. Tom's father brought a box of 75 pears to be distributed among the class. How many pears could Tom give to each of the 32 students? How many pears would be left over?
4. Mr. Jay distributed 2,457 blotters, dividing them evenly among 3 schools. How many did each school get?

Apparent quotient figures; check

5. On page 13 explain the dividing for $368 \div 7$ (box A).

To check a division example, we multiply the quotient and the divisor and add any remainder. Why should the answer be equal to the dividend? Explain the check in box B, page 13.

[W]

Divide and check your work in the following examples. When there is a remainder, write it beside the quotient with R.

6. $6 \overline{)468}$

7. $7 \overline{)2,464}$

8. $51 \overline{)765}$

9. $82 \overline{)9,486}$

10. $44 \overline{)7,968}$

11. $212 \overline{)7,420}$

12. $302 \overline{)8,775}$

13. $231 \overline{)9,982}$

➤ **Extra Practice.** Work Sets 17–19.

A	B
TensOnes 5 2 quotient 3 6 8 divisor 7 dividend 3 5 (5 tens $\times$ 7) 1 8 1 4 (2 $\times$ 7) 4 remainder	Check 52 quotient 364 $\times$ 7 divisor +4 remainder 368 dividend

What We Do in Dividing 368 by 7	What We Think
First Partial Dividend (getting tens) <i>Divide.</i> Use 36 (tens) as the first partial dividend. $36 \div 7 = 5$ and some over, so 5 is the first quotient figure. We write "5" in ten's place in the quotient. <i>Multiply</i> 7 by 5 (tens) and write the product, 35 (tens), under the 36 (tens). Then <i>compare</i> 35 with 36. The product of the first quotient figure and the divisor must not be greater than the first partial dividend. <i>Subtract.</i> 36 (tens) $-$ 35 (tens) = 1 (ten), the remainder. <i>Compare</i> the remainder with the divisor. The remainder figure (1) is smaller than the divisor (7). <i>Bring down.</i> 1 ten is part of the difference between 368 and 350. Bring down the other part (8 ones), forming the second partial dividend, 18 ones. Second Partial Dividend (getting ones) <i>Divide.</i> $18 \div 7 = 2$ and some over. We write "2" in one's place in the quotient. <i>Multiply</i> the 7 by 2 (ones) and write the product, 14, under 18. Then <i>compare</i> 14 with 18. 14 is not greater than 18. <i>Subtract.</i> 18 (ones) $-$ 14 (ones) = 4 (ones). Then <i>compare</i> the remainder with the divisor. The quotient is 52, and the remainder is 4.	$36 \div 7 = 5.$ $5 \times 7 = 35.$ 35 can be subtracted from 36. $36 - 35 = 1.$ 1 is less than 7. Bring down 8. $18 \div 7 = 2.$ $2 \times 7 = 14.$ 14 can be subtracted from 18. $18 - 14 = 4.$ 4 is less than 7. Answer: 52, R4

Some Hard Spots in Division

Non-apparent quotient figures; 0 in quotient [0]

Look at the work in the boxes below and answer the questions which follow.

A	B	C	D
$\begin{array}{r} 24, R32 \\ 39 \overline{)968} \\ \underline{78} \\ 188 \\ \underline{156} \\ 32 \end{array}$	$\begin{array}{r} 308 \\ 27 \overline{)8,316} \\ \underline{81} \\ 216 \\ \underline{216} \end{array}$	$\begin{array}{r} 40 \\ 84 \overline{)3,419} \\ \underline{336} \\ 59 \end{array}$	$\begin{array}{r} 500 \\ 76 \overline{)38,000} \\ \underline{380} \\ 00 \end{array}$

1. In box A, what is the first partial dividend? When we think, "How many 30's in 90?" or "How many 3's in 9?" we get 3 for the first quotient figure. Is 3 too large? How can you tell? Then try the next smaller number. How do you know that 2 is correct? Explain the rest of the work.

If a trial quotient figure is too large, try the next smaller number. Sometimes you will have to try more than once.

2. In box B, what is the second partial dividend? What is the quotient for $21 \text{ (tens)} \div 27$? Why must we write a zero as the second quotient figure? What do we do next? What is the third partial dividend?

3. In box C, what is the second partial dividend? How do we show that there are no 84's in 59 (ones)? What is the remainder?

4. In box D, why do we write zeros in ten's place and in one's place in the quotient?

After the first figure in the quotient, there must be 0 or some other figure above each figure of the dividend.

Practice in Dividing

[W]

Copy and work. Express any remainder with R.

- | a | b | c | d |
|----------------------------|-------------------------|------------------------|-------------------------|
| 1. $9\overline{)44}$ | $8\overline{)54}$ | $9\overline{)41}$ | $6\overline{)52}$ |
| 2. $7\overline{)5,215}$ | $8\overline{)3,430}$ | $7\overline{)9,021}$ | $6\overline{)5,070}$ |
| 3. $24\overline{)98,956}$ | $32\overline{)16,978}$ | $42\overline{)79,842}$ | $87\overline{)35,061}$ |
| 4. $18\overline{)4,973}$ | $46\overline{)2,645}$ | $17\overline{)15,130}$ | $28\overline{)21,380}$ |
| 5. $281\overline{)95,710}$ | $240\overline{)94,080}$ | $182\overline{)9,024}$ | $120\overline{)94,060}$ |
| 6. $4\overline{)61,948}$ | $9\overline{)41,680}$ | $3\overline{)10,000}$ | $5\overline{)8,000}$ |

► **Extra Practice.** If you made errors in any row above, do the first two rows of examples in the set indicated below.

For errors in rows	1	2	3	4	5	6
Work Set	17	18	19	20	21	22

For more practice in division, work Sets 23 and 24.

What Is n ? No Pencils, Please!

[O]

Find n , the missing number, without pencil and paper.

- $5 + 4, \div 3, \times 4, + 3, \div 5, - 2, + 9, \times 4 = n$
- $8 - 3, \times 5, + 5, \div 2, - 3, \div 3, \times 4, + 6 = n$
- $6 \times 3, - 3, + 6, \div 3, \times 2, + 6, + 8, \div 7 = n$
- $9 \div 3, + 4, \times 3, + 4, \div 5, + 7, + 6, \div 9 = n$
- $7 + 6, - 5, \times 4, + 3, \div 5, \times 3, + 4, \div 5 = n$
- $9 - 5, + 4, \times 3, + 6, \div 2, \div 3, \times 2, + 10 = n$
- $4 \times 7, + 2, \div 2, \div 3, \times 4, + 1, \div 7, + 4 = n$



Can You Work with Averages?

[W]

Jack, Bob, Bill, and Sam went on an overnight hike to Frost Mountain.

1. Before starting, the boys weighed their packs to see how much they would carry. The packs weighed as follows: Jack's, 25 lb.; Bob's, 35 lb.; Bill's, 20 lb.; and Sam's, 32 lb. What was the average weight each boy carried?

- Must you first find the total weight of the packs?
- What fractional part of the total gives the average?

2. The boys rode their bicycles to Bill's house and went by bus from there. To reach Bill's house, it took Jack 16 min., Bob 20 min., and Sam 30 min. What was their average time for reaching Bill's house?

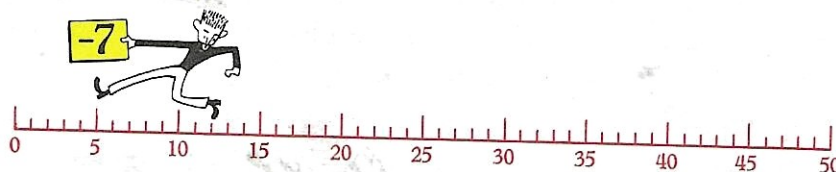
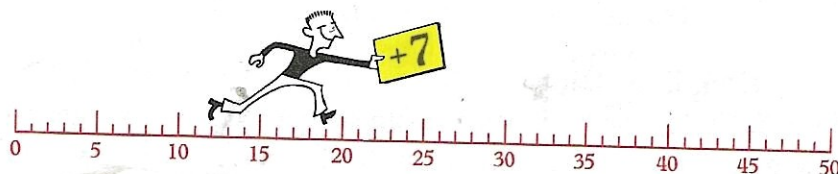
3. Each boy took eggs for breakfast. Jack took 3, Bob 5, Bill 4, and Sam 4. What was the average number of eggs carried by a boy?

4. The four boys took an average of \$2.75 apiece for spending money. Then among them, the boys had how much money?

Using a Number Line

A., S., M., and D. [O]

The use of a number line often helps us to understand the operations of arithmetic. By using a number line, we can see that **we are putting groups together when we add or multiply and taking a group apart when we subtract or divide.**



1. To **add** 12 and 7 on the number line, first count 12 spaces. Then **move to the right** 7 more spaces. To what point does this bring you? Is 19 the sum of 12 and 7?

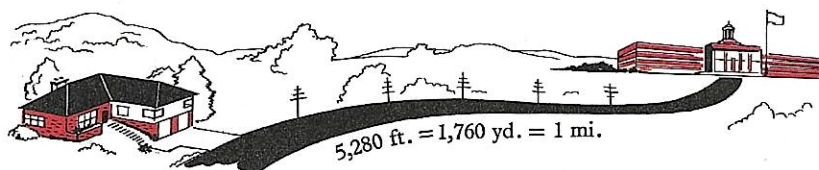
2. To **subtract** 7 from 12, start at the end of 12 spaces and **move to the left** 7 spaces. $12 - 7 = ?$

3. To **multiply** 4 by 5, start at 0. **Move to the right** 4 spaces; then 4 more; then 4 more; then 4 more. Have you laid off 5 groups of 4? What point on the line have you reached? $5 \times 4 = ?$

4. To **divide** 18 by 3, start at the end of 18 spaces. **Move to the left** as many groups of 3 as possible and count the number of groups. Are there 6? $18 \div 3 = ?$

Use the number line in finding answers for the following:

- | | | | |
|------------------|----------------|-------------------|------------------|
| 5. $18 + 4$ | 6. $24 \div 8$ | 7. 6×6 | 8. $35 - 9$ |
| 9. 3×13 | 10. $12 + 15$ | 11. $19 - 13$ | 12. $28 \div 4$ |
| 13. $14 + 29$ | 14. $36 - 17$ | 15. 3×14 | 16. $44 \div 11$ |



Round Numbers Can Help You!

Using large numbers; rounding; estimating [O]

1. Dick lives 1 mile from school. We might say that he lives 5,280 feet or 1,760 yards from school. Why do we generally give the distance in miles instead of in feet or in yards?

2. The price of a coat was 47 dollars. It was also 470 dimes, or 4,700 cents. Why would a store give the price in dollars instead of in dimes or in cents?

3. Which amount is hardest to think—3 million dollars or 3,000,000 dollars or 30,000,000 dimes or 300,000,000 cents? Why are the values of all these amounts equal?

4. Why do newspapers print \$3,200,000,000 as \$3.2 billion?

It is easier to understand an amount if it is expressed in the largest unit that is appropriate.

5. A city with a population of 400,000 spends an average of \$30 a year per person for its public schools. What is the quickest way of finding the total spent per year?

6. For Ex. 5, do you think that the population of the city is exactly 400,000? Why don't we use an exact number?

Often it is better to use an approximate number—that is, one that tells us *about* how many. The population at the time of the last census might have been a little more or less than 400,000 but, in round numbers, we think "400,000."

It is often convenient to use round numbers when we are making estimates or are talking about large numbers or distances or quantities that change often.

Numbers may be rounded to the nearest ten, hundred, thousand, and so on, depending on how exact we wish to be. A number just halfway between two numbers is generally rounded to the next larger number.

7. If we want to round to the nearest ten, we round 487 to $__?$; 483 to $__?$; and 485 to $__?$. Explain why.

[W]

Round to the nearest ten; and then to the nearest hundred.

8. 444 9. 176 10. 969 11. 345 12. 3,077
13. 405 14. 1,641 15. 894 16. 732 17. 555

Round numbers can help you in working problems. Suppose that, in working a problem, you needed to find the total of the numbers shown in the left-hand column in the box. By rounding the numbers to thousands, as shown in the right-hand column, you could quickly find an approximate answer.

	Estimate
10,049	10,000
1,963	2,000
8,102	8,000
<u>+7,946</u>	<u>+8,000</u>
?	28,000

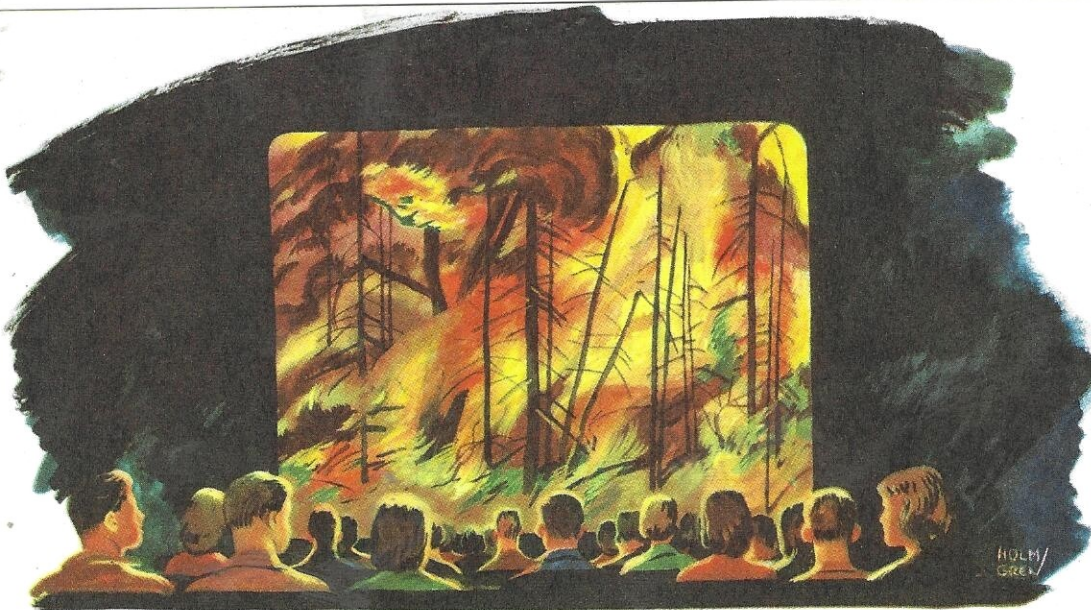
18. Find the exact answer by adding the numbers in the left-hand column. Is the estimate a good one?

In working an example or problem, it is often a good idea to round the numbers and estimate the answer before you try to find the exact answer. Then you can test your answer by comparing it with the estimate to see whether it makes sense.

In working the following, first round the numbers and write your estimated answer. Then find the exact answer and compare it with your estimate to see if your answer is sensible.

19. $\begin{array}{r} 70,956 \\ +12,139 \end{array}$ 20. $\begin{array}{r} 96,048 \\ -8,865 \end{array}$ 21. $\begin{array}{r} 397 \\ \times 21 \end{array}$ 22. $\begin{array}{r} 819 \\ \times 39 \end{array}$ 23. $\begin{array}{r} 19 \overline{)602} \end{array}$

➤ **Extra Practice.** Work Set 25.



Conserving Our Natural Resources

Differentiating A., S., M., D.; estimating the answer [W]

Because of our poor conservation practices, much good timber and topsoil in our land has been lost through fire and through erosion (the wearing action of wind and water).

In each of the following problems, first round the numbers given in the problem and write down an estimated answer. Then find the exact answer to the problem and test it by your estimate.

1. Jim's class saw a motion picture showing how the Forest Service fights fires. One forest fire that was discovered at 3:55 P.M. on Tuesday was finally brought under control at 4:30 A.M. on Thursday. How many hours was the fire out of control after it was discovered?

2. In a recent year, 7,283 forest fires west of the Rocky Mountains caused losses averaging \$1,435 per fire. What was the total loss from these fires?

3. In the United States there are 151 national forests totaling 181,255,449 A. (acres). Find the average number of acres per national forest.

4. If furrows on a hill are plowed up and down and not according to the contour of the slope, the land may lose as much as 3,200 lb. of soil per acre in 6 mo. through erosion by rain. How many tons of soil could 10 acres lose in 6 mo.? (2,000 lb. = 1 ton.)

5. Mr. Rowe bought 42 A. of worn-out farm land for \$15 an acre. By using scientific methods of farming, he improved the land so much that in 10 years it was valued at \$5,450. How much had the land increased in value in the 10 years?

6. Mr. Brown owned 200 A. of timber land. He was offered \$2,850 for all the trees on this land. Instead, Mr. Brown thinned his woods with the forester's help. He sold \$5,925 worth of trees for lumber and \$4,212 worth for firewood. How much more did Mr. Brown make by thinning his woods than he would have made by selling all the trees standing on his land?

7. Mr. Hill hired boys to set out seedling pines on 37 A. of worn-out pasture land. He figured that he would need 1,050 seedlings per acre. How many did he need in all?



Finding Errors

[W]

In solving problems, you cannot get correct answers if you make mistakes.

These examples are from Paul's paper. Some are wrong. Work each example and compare your work with Paul's.

1. $\begin{array}{r} 496 \\ \times 8 \\ \hline 4,058 \end{array}$	2. $\begin{array}{r} 98 \\ \times 700 \\ \hline 68,600 \end{array}$	3. $\begin{array}{r} 76,097 \\ \times 8 \\ \hline 588,776 \end{array}$	4. $\begin{array}{r} 7,690 \\ \times 400 \\ \hline 307,600 \end{array}$
---	---	--	---

5. $\begin{array}{r} 407 \\ \times 63 \\ \hline 1\ 221 \\ 24\ 42 \\ \hline 25,641 \end{array}$	6. $\begin{array}{r} 804 \\ \times 72 \\ \hline 168 \\ 56\ 28 \\ \hline 56,446 \end{array}$	7. $\begin{array}{r} 327 \\ \times 52 \\ \hline 654 \\ 16\ 35 \\ \hline 17,004 \end{array}$	8. $\begin{array}{r} 431 \\ \times 23 \\ \hline 1\ 293 \\ 862 \\ \hline 2,155 \end{array}$
--	---	---	--

9. $\begin{array}{r} 81 \\ 7 \overline{)567} \end{array}$	10. $\begin{array}{r} 212 \\ 3 \overline{)636} \end{array}$	11. $\begin{array}{r} 10 \\ 42 \overline{)420} \end{array}$	12. $\begin{array}{r} 233 \\ 6 \overline{)1,428} \end{array}$
---	---	---	---

13. $\begin{array}{r} 32, R13 \\ 12 \overline{)397} \\ \underline{36} \\ 37 \\ \underline{24} \\ 13 \end{array}$	14. $\begin{array}{r} 315 \\ 25 \overline{)7,875} \\ \underline{75} \\ 37 \\ \underline{25} \\ 125 \\ \underline{125} \end{array}$	15. $\begin{array}{r} 19, R36 \\ 67 \overline{)7,339} \\ \underline{67} \\ 639 \\ \underline{603} \\ 36 \end{array}$
--	--	--

Practice with Whole Numbers

Copy and find the answers.

[W]

	a	b	c	d	e	f
1.	$\begin{array}{r} 843 \\ 23 \\ 78 \\ +506 \\ \hline \end{array}$	$\begin{array}{r} 135 \\ 87 \\ 249 \\ +901 \\ \hline \end{array}$	$\begin{array}{r} 531 \\ 94 \\ 109 \\ +682 \\ \hline \end{array}$	$\begin{array}{r} 7,943 \\ -2,086 \\ \hline \end{array}$	$\begin{array}{r} 4,132 \\ -2,008 \\ \hline \end{array}$	$\begin{array}{r} 5,174 \\ -906 \\ \hline \end{array}$
2.	$\begin{array}{r} 709 \\ \times 697 \\ \hline \end{array}$	$\begin{array}{r} 987 \\ \times 354 \\ \hline \end{array}$	$\begin{array}{r} 182 \\ \times 309 \\ \hline \end{array}$	$27 \overline{)2,691}$	$69 \overline{)2,901}$	$34 \overline{)4,430}$

Finding n , the Missing Number

Addition and subtraction [O]

A		Parts	Total	
a.	$853 + 271 = 1,124$			To find the missing number,
b.	$853 + n = 1,124$			$\rightarrow n = 1,124 - 853$
c.	$n + 271 = 1,124$			$\rightarrow n = 1,124 - 271$

B		Total	Parts	
a.	$1,124 - 271 = 853$			To find the missing number,
b.	$1,124 - n = 853$			$\rightarrow n = 1,124 - 853$
c.	$1,124 - 271 = n$			$\rightarrow n = 1,124 - 271$

When you are solving addition and subtraction problems, you will need to find a missing number, but you may not know whether to add or to subtract.

1. In box A, Ex. a, what are the addends? What is the sum?
2. In box B, why is the number from which we subtract really the total? Why are the number subtracted and the number left over really parts of the total? What names do we use for the parts and the total of a subtraction example?
3. For any addition or subtraction example, if you know the total and one part, how can you find the missing part?

[W]

In these A. and S. examples, find n , the missing number.

a	b	c
4. $821 + n = 914$	$1,572 - n = 695$	$21,471 + n = 85,003$
5. $934 - n = 257$	$2,831 - 1,976 = n$	$n + 45,259 = 72,015$
6. $485 - 239 = n$	$4,249 + n = 5,180$	$49,278 - n = 35,763$
7. $n + 581 = 732$	$n + 3,001 = 8,000$	$n + 53,108 = 93,217$
8. $395 + n = 684$	$n + 5,489 = 7,302$	$5,875 - n = 217$
9. $731 - 365 = n$	$8,493 - n = 2,579$	$75,000 - 823 = n$

When we know a total and one of its two parts, we subtract to find the missing part.



Can You Ask the Missing Question?

Each member of the class agreed to earn money during the summer for the Athletic Fund. Here are some of their schemes. ^[W]
Copy each exercise, write a question, and solve the problem.

1. Polly washed dishes in a restaurant. She worked 2 hours a day at 60¢ an hour for eight 6-day weeks.
2. Linda and Sally picked a 10-quart pail of blueberries and sold them to the fruit man at 25¢ a quart.
3. Jim had a radio rental service. He paid \$24 for a small radio and rented it for \$3 a week for 14 weeks.
4. Bill found some catnip growing near his house. He sold 25 bunches of it to the owner of a pet shop for 6¢ a bunch. The pet shop's price to customers was 10¢ a bunch.
5. The owner of a riding stable pays Joe 50¢ an hour to help him groom and exercise the horses. Joe worked 3 hours a day each day except Sunday for 12 weeks.
6. Ed and Tom ran an "Errand Service." They had 12 customers who each paid \$1.50 a week to have their mail collected twice a day and errands done at two stores.

Do You Understand?

Test of Information and Meaning 1

1. From the following, make a list of cardinal numbers and a list of ordinal numbers:

a. 17 dogs; b. May 31; c. 218 Main Street; d. 5,280 ft.;
e. room 29; f. 495; g. 75th; h. 4,075 people.

2. Write the meaning of 87,290,541, figure by figure.

3. Which process is best to use for finding the total of each of the following?

a. $78 + 78 + 78 + 78 = ?$ b. $29 + 31 + 72 + 46 = ?$

4. Why is 12 not a prime number?

5. In the example $9,005 - 1,897$, from what do you borrow 1 ten?

6. In the example $24 \overline{)8,351}$, what is the first partial dividend?

7. Are 2, 10, and 4 the prime factors of 80? Why?

8. In 49×86 , what is the second partial product?

9. Is the following a problem in measurement division or fractional-part division?

Sam's father divided \$10 equally among his 5 boys. How much did each boy receive?

10. Round each of the following to the nearest ten:

a. 759; b. 1,875; c. 2,532.

11. Round to the nearest hundred the numbers in Ex. 10.

12. Draw a number line showing the distance from 0 to 40. On it show how to find the quotient for $39 \div 13$.

13. To multiply 75 by 823, what example would you write?

14. Write the number which has 3 in one's place and 6 in thousand's place with zeros in the other places.

15. Write in Roman numerals the number 1955.

Do You Make Mistakes?

Diagnostic Test 1

Copy, and find and check answers. Show any remainder in division with R.

- | | | | | | |
|----|------------|------------|-------------|---------------|-----------------|
| | a | b | c | d | e |
| 1. | 29 | 41 | 628 | 8,249 | 486,594 |
| | 37 | 87 | 357 | <u>+7,586</u> | 142,737 |
| | 83 | 94 | 109 | | <u>+394,151</u> |
| | 65 | 72 | 320 | | |
| | <u>+49</u> | <u>+25</u> | <u>+506</u> | | |
| 2. | 83 | 72 | 908 | 817 | 870 |
| | <u>-65</u> | <u>-59</u> | <u>-699</u> | <u>-748</u> | <u>-205</u> |
| 3. | 27 | 417 | 48 | 359 | 2,935 |
| | <u>×5</u> | <u>×9</u> | <u>×58</u> | <u>×72</u> | <u>×49</u> |

4. $6\overline{)4,382}$ $45\overline{)832}$ $74\overline{)2,367}$ $16\overline{)3,894}$ $183\overline{)1,739}$

If you made errors in	Row 1	Row 2	Row 3	Row 4
Study text pages	6-7	10-11	6-7	13-14
Then work Set	4	9	15	24

How Well Can You Figure?

Computation Test 1

Copy, work, and check. Show division remainders with R.

- | | | | | | | |
|----|------------|-----------------------|------------------------|-----------------------|-------------|-----------------|
| | a | b | c | d | e | f |
| 1. | 93 | 62 | 4,583 | | 756 | 67,348 |
| | <u>-89</u> | <u>×34</u> | <u>+497</u> | $75\overline{)7,650}$ | <u>-468</u> | <u>+19,527</u> |
| 2. | 738 | 500 | | 516 | 4,526 | 629,215 |
| | <u>×49</u> | <u>-89</u> | $736\overline{)6,148}$ | <u>×4</u> | <u>×637</u> | 871,543 |
| | | | | | | <u>+135,205</u> |
| 3. | 54 | | 484 | | 39 | |
| | <u>-38</u> | $17\overline{)6,183}$ | <u>-275</u> | $8\overline{)5,214}$ | <u>×7</u> | |

Can You Solve Problems?

Problem Test 1

Write your work for the following. Check each solution to make sure that you are working accurately.

1. For his vacation trip, Jack Harmon bought a suitcase for \$16.85; new slacks for \$4.98; and a sweater for \$5.95. How much did he spend for these things?

2. Jack and his friend Henry decided to take a motor trip through the mountains and looked up mileage routes. The first day's trip by way of Overland would be 386 miles and by way of Westland, 449 miles. How much shorter was the route by way of Overland than by way of Westland?

3. When they started, Jack made a record that the speedometer reading was 14,285 mi. After the first day's trip by way of Westland, what mileage did the speedometer reading show?

4. Henry kept a record of the gasoline they bought on the trip. On the 1st day they bought 12 gal., 8 gal., 4 gal.; on the 2d day, 12 gal., 6 gal., 3 gal.; on the 3d day, 10 gal., 8 gal., 5 gal. How much gasoline did the boys buy during their first three days?

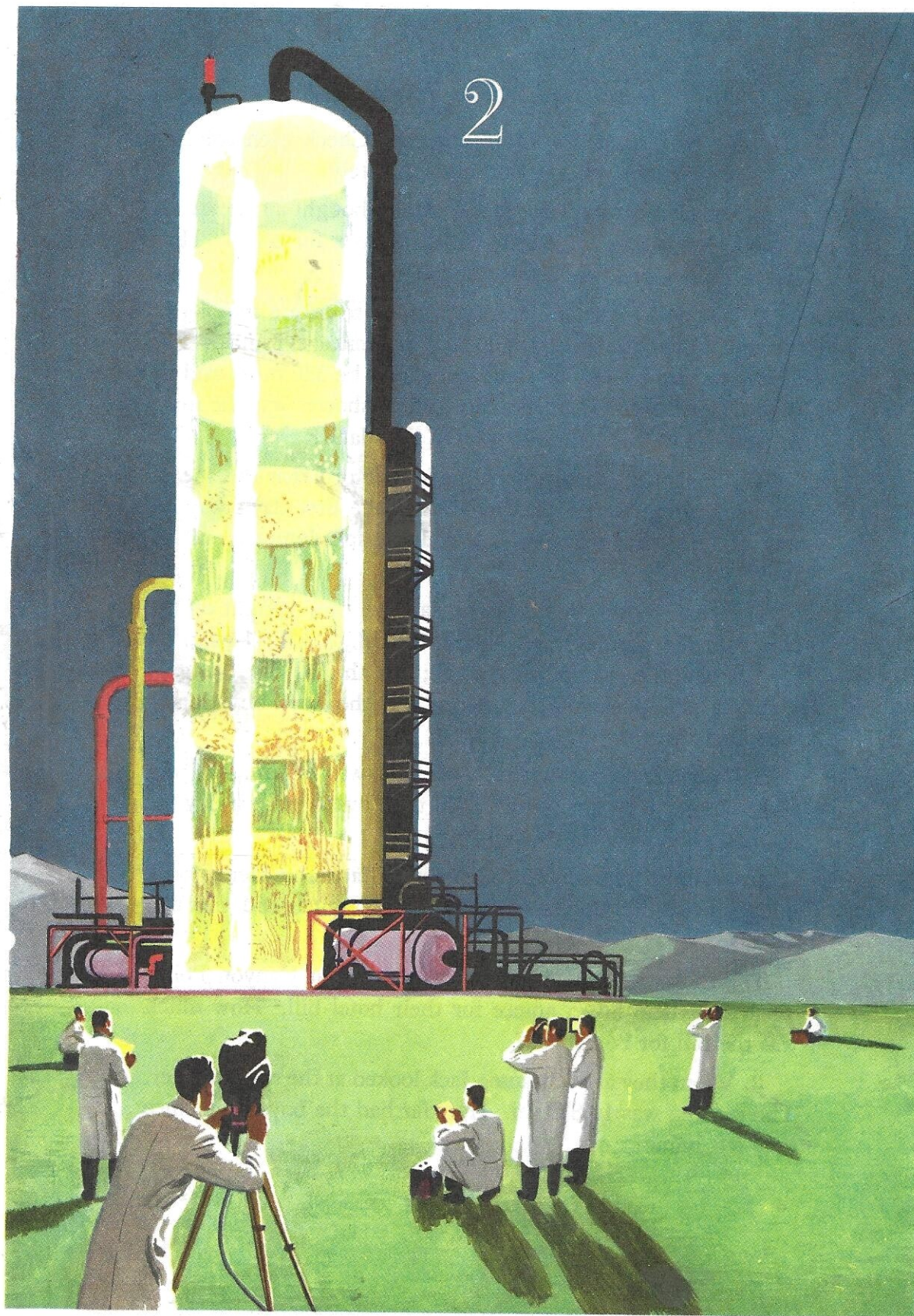
5. The mileage for the first three days was 449 mi., 424 mi., and 351 mi. How many miles did the boys average a day for the first three days?

6. At the Mountain Inn Gift Shop, Jack and Henry both bought souvenir jackets at \$4.90. How much did the two jackets cost?

7. The boys decided to stay over a day at Canyon Point. That cost them \$8.75 apiece for their hotel bill. How much was the bill for both boys?

8. When they arrived home Jack looked at the speedometer. The reading was 16,879 mi. How far had the boys been since they left home?

2



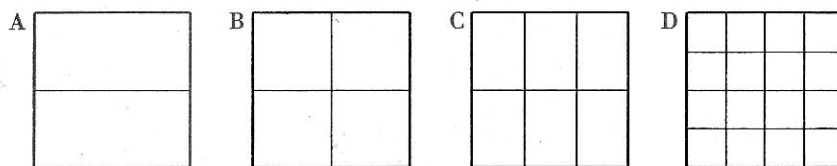
How Are Fractions Used?

Inventory Test [W]

Perhaps you have thought about fractions as a subject in a book studied during arithmetic period. The picture on the opposite page shows a tower used for fractionating petroleum. In this fractionating tower, crude oil is broken up by heat into its parts, called fractions. These fractions are gasoline, kerosene, fuel oil, lubricating oil, and so on. The fractional parts of the oil are not necessarily equal parts.

Work the following problems to see if you know how to use fractions:

1. Each person at the study table put away $\frac{1}{5}$ of the 120 books. How many books was that apiece?
2. Mr. Spencer said that $\frac{5}{8}$ of the 204 students in the assembly hall do not go home during the noon hour. How many students do not go home during the noon hour?
3. Jack, Sam, and Henry each ate $\frac{1}{4}$ of a pie. How much of the pie was left?
4. Sally said that her average monthly weight during the last 6 months was $75\frac{1}{8}$ lb. This was $2\frac{1}{2}$ lb. more than her monthly average for the preceding 6 months. What was her monthly average during the preceding 6 months?
5. The Hazen family toured the West for $2\frac{1}{2}$ months. With 4 weeks to a month, how many weeks did the Hazens spend touring?
6. Many people sleep $\frac{1}{3}$ of every 24 hours. How long is that?
7. Each reel of wire cable used in the construction of a transmission line holds about $\frac{3}{4}$ mi. of cable. About how many feet of cable are there on a reel?
8. Scientists have figured out that a space station for interplanetary travel will move in its path around the earth at the rate of $4\frac{2}{5}$ mi. per second. How many miles an hour will the space station move?



What Are Fractions?

Meaning; terms; fractional unit [W]

Squares A, B, C, and D, shown above, are equal in size. Square A is divided into 2 equal parts. Each of the parts is $\frac{1}{2}$ of square A.

1. Square B is divided into how many equal parts? Each of the parts is $\frac{1}{?}$ of square B.

2. Square C has $_{?}$ equal parts. Each of the equal parts is $\frac{1}{?}$ of square C.

3. Square D has $_{?}$ equal parts. Each of the equal parts is $\frac{1}{?}$ of square D.

4. The **fractional unit** for square A is $\frac{1}{2}$; for square B is $_{?}$; for square C is $_{?}$; for square D is $_{?}$.

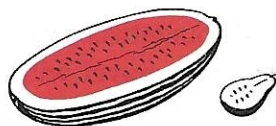
In the fraction naming the fractional unit, the numerator (the number above the line) is always 1.

The denominator (the number below the line) shows the number of equal parts in the whole and so it indicates the size of the fractional unit.

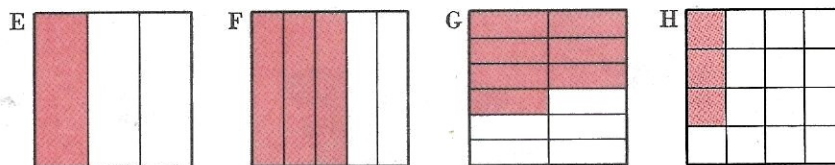
5. Look again at the squares at the top of the page. As the number of equal parts is increased, what happens to the size of the fractional unit? to the denominator of the fraction indicating the fractional unit?

6. Which is larger, $\frac{1}{3}$ or $\frac{1}{6}$ of the same whole? $\frac{1}{14}$ or $\frac{1}{7}$? $\frac{1}{5}$ or $\frac{1}{12}$? $\frac{1}{9}$ or $\frac{1}{8}$?

7. Why is $\frac{1}{2}$ of a watermelon larger than $\frac{1}{2}$ of a pear?



8. Name two $\frac{1}{4}$'s which are not equal in size.



Multiple fractions [O]

9. In square E above, what is the fractional unit? How many equal parts of square E are colored? How many parts are not colored?

10. Answer the same questions for square F; square G; and square H.

11. Write on the board the fraction that stands for the part not colored in square E; in square F; in square G; in square H. What does the numerator show in each case? the denominator?

A fraction is a number and means one or more equal parts of the size indicated by the denominator.

12. $\frac{4}{5}$ means "four $\frac{1}{5}$'s." What do these fractions mean:

$\frac{3}{8}$? $\frac{7}{10}$? $\frac{5}{12}$?

13. If the wholes are equal, which fraction is larger:

$\frac{2}{3}$ or $\frac{2}{12}$? $\frac{3}{16}$ or $\frac{3}{5}$? $\frac{7}{16}$ or $\frac{7}{12}$? $\frac{5}{12}$ or $\frac{5}{16}$?

(Look at the squares above if you need to.)

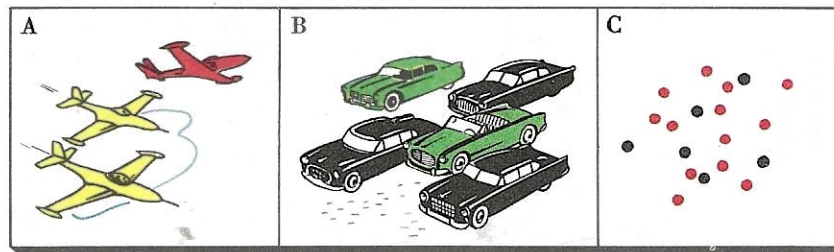
14. When the wholes are equal, which fraction is smaller:

$\frac{1}{8}$ or $\frac{2}{3}$? $\frac{5}{12}$ or $\frac{7}{12}$? $\frac{2}{5}$ or $\frac{4}{5}$? $\frac{7}{16}$ or $\frac{9}{16}$?

Do you remember these **fraction laws**? Explain them.

I. When wholes are the same size and the fractions have equal numerators, the fraction with the largest fractional unit (that is, with the smallest denominator number) has the greatest value.

II. When wholes are the same size and the fractions have fractional units which are like, the fraction having the largest numerator has the greatest value.



A Fraction of a Group

Meaning [O]

1. In box A, when one plane leaves the group, what part of the group is breaking away?
2. What fraction of the group of cars in picture B is black?
3. In box C, what fraction of all the dots is red?

A fraction may mean one or more of the equal parts of a group of things.

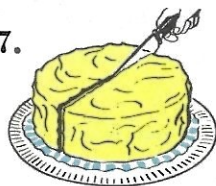
[W]

Give the best answers you can for the following:

4. Make drawings to show $\frac{7}{8}$ of a square and also $\frac{7}{8}$ of a bunch of roses.
5. Draw a line and make $\frac{5}{8}$ of it darker than the rest of it.
6. Make any drawings you wish in order to illustrate $\frac{13}{16}$ of a group and $\frac{13}{16}$ of an object.

On your paper, write “fraction of object” or “fraction of group” to tell for each of the following what the drawing shows:

7.



8.



9.



Fractions in the Number System

In our modern world, where so many things depend on accurate measurement, it would be impossible to get along without fractions. ^[O]

1. Tell ways in which the invention of fractions has made the number system much more useful than if it contained only whole numbers.

2. In the number 6,213 the 3 is in $_{-?}$ place, so we know that it means 3 ones, or 3 wholes. Its place in the number tells us its value. What does the 3 mean in 3,748? in 9,436? in 4,396?

3. What does the 3 tell us in the number $\frac{3}{7}$?

A fraction, besides being a number, may mean other things too.

4. Mr. Keldon bought 4 dozen oranges and distributed them among the families of his three children. How many dozen oranges did each family receive?

$$\begin{array}{r} 1\frac{1}{3} \\ 3 \overline{)4} \\ \underline{3} \\ 1 \end{array}$$

In the box, what does the fraction mean in the quotient? Why does $\frac{1}{3}$ mean $1 \div 3$?

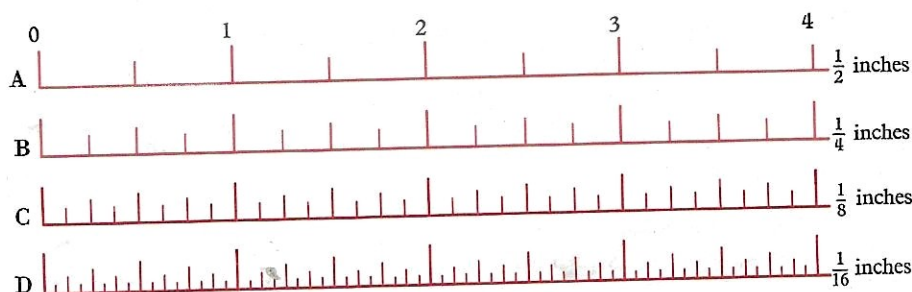
Any fraction may be thought of as an indicated division.

Tell the meaning of the indicated divisions in Ex. 5–10.

5. $\frac{7}{8}$ 6. $\frac{11}{15}$ 7. $8\frac{3}{5}$ 8. $10\frac{1}{2}$ 9. $\frac{15}{7}$ 10. $\frac{9}{20}$

Write the fraction to show an indicated division which means ^[W]

11. 3 (apples) divided among 4 (people).
12. 17 (doughnuts) divided among 8 (people).
13. 45 (yards of bunting) divided among 10 (rooms).
14. 750 (dollars) divided among 1,000 (students).



Equivalent Fractions

Meaning; using number line [O]

Using lines A-D, give the answers to the following:

1. $\frac{1}{4} = \frac{?}{8}$
2. $\frac{6}{8} = \frac{?}{4}$
3. $\frac{8}{16} = \frac{?}{2}$
4. $\frac{5}{8} = \frac{?}{16}$
5. $\frac{1}{4} = \frac{?}{16}$
6. $1\frac{1}{2} = \frac{?}{4}$
7. $\frac{5}{4} = \frac{?}{16}$
8. $\frac{2}{8} = \frac{?}{16}$
9. $\frac{7}{8} = \frac{?}{16}$
10. $1\frac{2}{8} = \frac{?}{4}$
11. $1\frac{2}{16} = \frac{?}{8}$
12. $2 = \frac{?}{8}$

Study Ex. 13-15 to find how to work Ex. 1-12 by using numbers instead of lines.

13. When we **multiply both the numerator and the denominator** of the fraction $\frac{1}{2}$ by 1, that is, when $\frac{1}{2}$ is multiplied by $\frac{1}{1}$, do we change the value of $\frac{1}{2}$? $\frac{1 \times 1}{1 \times 2} = \frac{1}{2}$

14. Why do we not change the value of the fraction $\frac{1}{2}$ when we multiply both the numerator and the denominator by 4? $\frac{4 \times 1}{4 \times 2} = ?$ Check by the lines above to see if $\frac{4}{8} = \frac{1}{2}$.

15. When we **divide both the numerator and the denominator** of $\frac{6}{8}$ by 2, that is, when the fraction is divided by $\frac{2}{2}$, why do we not change the value of the fraction $\frac{6}{8}$? Check by the lines above to see if $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$.

[W]

16-27. Work Ex. 1-12 by using numbers.

If both the numerator and the denominator of a fraction are multiplied (or divided) by the same number, the value of the fraction is not changed.

Fractions having the same value are equivalent.
Equivalent fractions may have different fractional units.

Practice with Equivalent Fractions

Copy and complete the following:

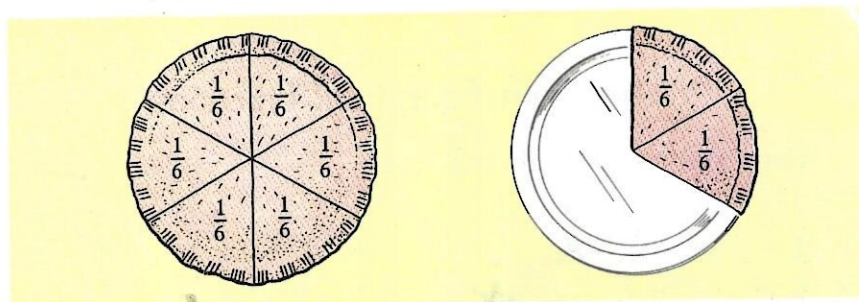
[W]

- | | a | b | c | d | | a | b | c | d |
|-----------------------|----------------|----------------|-----------------|----------------|-----------------------|---------------|----------------|----------------|----------------|
| 1. $\frac{1}{4} =$ | $\frac{?}{16}$ | $\frac{?}{8}$ | $\frac{?}{64}$ | $\frac{?}{32}$ | 6. $1 =$ | $\frac{?}{4}$ | $\frac{?}{13}$ | $\frac{7}{?}$ | $\frac{9}{?}$ |
| 2. $\frac{3}{5} =$ | $\frac{?}{10}$ | $\frac{?}{25}$ | $\frac{12}{?}$ | $\frac{18}{?}$ | 7. $2\frac{1}{2} =$ | $\frac{?}{2}$ | $\frac{?}{8}$ | $\frac{15}{?}$ | $\frac{?}{4}$ |
| 3. $\frac{50}{100} =$ | $\frac{?}{2}$ | $\frac{?}{20}$ | $\frac{100}{?}$ | $\frac{5}{?}$ | 8. $2 =$ | $\frac{?}{3}$ | $\frac{?}{4}$ | $\frac{12}{?}$ | $\frac{10}{?}$ |
| 4. $\frac{5}{8} =$ | $\frac{?}{40}$ | $\frac{?}{16}$ | $\frac{20}{?}$ | $\frac{15}{?}$ | 9. $\frac{8}{4} =$ | $\frac{?}{2}$ | $\frac{?}{1}$ | $\frac{16}{?}$ | $\frac{?}{12}$ |
| 5. $\frac{5}{4} =$ | $\frac{?}{8}$ | $\frac{?}{20}$ | $\frac{10}{?}$ | $\frac{40}{?}$ | 10. $\frac{12}{16} =$ | $\frac{?}{4}$ | $\frac{6}{?}$ | $\frac{?}{32}$ | $\frac{48}{?}$ |

Problems with fractions; measures

11. $\frac{7}{8}$ inch equals how many $\frac{1}{16}$ inches? ($\frac{7}{8} = \frac{?}{16}$)
12. How many ninths in $\frac{1}{3}$?
13. How many fifteenths in $\frac{1}{5}$?
14. How many half yards in $\frac{9}{8}$ yd.? ($\frac{9}{8} = \frac{?}{2}$)
15. $\frac{80}{100}$ equals how many fifths? how many twenty-fifths?
16. Some rulers are divided into $\frac{1}{16}$ -inch spaces. How many $\frac{1}{16}$ -inch spaces are there in a length of $\frac{5}{8}$ inch?
HINT: Do you see that the easy way to solve this problem is to change $\frac{5}{8}$ to 16ths?
17. A ribbon $\frac{2}{3}$ yd. long was cut into strips each $\frac{1}{8}$ yd. in length. How many strips were there?
18. A druggist put $\frac{3}{4}$ ounce of a prescription into capsules each of which held $\frac{1}{16}$ ounce. How many capsules were filled from the $\frac{3}{4}$ ounce?
19. A box of apples contains $2\frac{1}{2}$ bushels. How many $\frac{1}{4}$ -bushel baskets can be filled from the box?
20. Dick and Joe each bought a 60-cent book. Dick told his mother that the book cost $\frac{3}{5}$ of a dollar. Joe told his mother that the book cost $\frac{6}{10}$ of a dollar. Which of the boys was correct in the way he gave the cost?

➤ **Extra Practice.** Work Set 26.



Kinds of Fractions

Meaning; changing to best form [O]

1. The pie at the left, above, has been cut into equal parts. What is the fractional unit? There are $\frac{2}{6}$ in the whole pie.

2. The 2 pieces of a pie of the same size at the right equal $\frac{2}{6}$ of a pie. In all, how many sixths are shown above?

A fraction equal to or greater than one whole, such as $\frac{8}{6}$, is called an **improper fraction**. Any fraction meaning less than 1 whole, such as $\frac{2}{6}$, is called a **proper fraction**.

3. Since the diagram above shows $1\frac{2}{6}$ whole pie and $\frac{2}{6}$ of another pie, we could say that it shows $1 + \frac{2}{6}$, or $1\frac{2}{6}$, pies.

A whole number and a fraction added together make a **mixed number**.

4. To change an improper fraction, such as $\frac{12}{8}$, to a mixed number, we can *think*,

"From the denominator of the fraction, we know the number of equal parts in 1 whole ($1 = \frac{8}{8}$). In $\frac{12}{8}$, there are as many wholes as there are $\frac{8}{8}$ in $\frac{12}{8}$.

"A short way to find how many wholes in $\frac{12}{8}$ is to divide the numerator, 12, by the denominator, 8. $\frac{12}{8} = 12 \div 8$, or $1\frac{4}{8}$."

In working with fractions, we often get answers which are improper fractions, but we generally do not leave them in that form. **When an improper fraction has been changed to a whole number or to a mixed number, it has been changed to better form.**

5. Why are $1\frac{2}{6}$ (Ex. 3) and $1\frac{4}{8}$ (Ex. 4) still not in best form? Are the fractions $\frac{2}{6}$ and $\frac{4}{8}$ in **lowest terms**?

You know that the value of a fraction is not changed when both its terms (numerator and denominator) are divided by the same number. **When we divide by the largest possible number, we reduce the fraction to lowest terms. It is then in best form.**

6. Give $1\frac{2}{6}$ and $1\frac{4}{8}$ in best form.

In working with fractions, you will also need to change a mixed number to an improper fraction.

7. To change $2\frac{4}{7}$ to an improper fraction, we can *think*, "In 1 whole there are $\frac{7}{7}$, so in 2 wholes there are $2 \times \frac{7}{7}$, or $\frac{14}{7}$. Add the extra $\frac{4}{7}$ and we have in all $\frac{14}{7} + \frac{4}{7}$, or $\frac{18}{7}$."

A short way to change a mixed number to an improper fraction is to

multiply the denominator of the fraction by the whole number;
add the numerator of the fraction to that product;
write the sum over the denominator.

8. Change these improper fractions to whole numbers or to mixed numbers in best form: [W]

$\frac{9}{5}$; $\frac{7}{7}$; $\frac{12}{4}$; $\frac{15}{8}$; $\frac{21}{4}$; $\frac{19}{10}$; $\frac{11}{3}$; $\frac{13}{12}$.

9. Change these mixed numbers to improper fractions:

$7\frac{1}{8}$; $4\frac{4}{5}$; $10\frac{5}{7}$; $3\frac{11}{12}$; $16\frac{2}{3}$; $11\frac{1}{9}$; $21\frac{1}{2}$; $6\frac{3}{4}$.

In these exercises, find the fractions not in best form and write them on your paper. Then change them to best form.

	a	b	c	d	e	f
10.	$\frac{4}{6}$	$\frac{1}{3}$	$\frac{7}{12}$	$\frac{6}{5}$	$\frac{4}{4}$	$\frac{2}{3}$
11.	$\frac{9}{12}$	$\frac{15}{3}$	$\frac{1}{7}$	$\frac{9}{5}$	$\frac{22}{24}$	$\frac{4}{5}$
12.	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{9}{10}$	$\frac{12}{10}$	$\frac{23}{10}$

► **Extra Practice.** Work Sets 27–30.

Adding and Subtracting Fractions

Like- and unlike-fractions; common denominator [O]

When we add 9 books plus 7 books, the "9 plus 7" tells how many are added and the "books" tells what kind of things are added. In the same way, if we add 9 sixteenths plus 7 sixteenths, the "9 plus 7" tells how many are added and the "sixteenths" tells what kind of equal parts are added.

1. In the example $\frac{2}{7} + \frac{4}{7} + \frac{3}{7}$, which numbers tell how many are added? Which tell what kind of equal parts are added?

2. $4 \text{ ninths} + 8 \text{ ninths} + 7 \text{ ninths} = ?$

3. $\frac{7}{15} + \frac{9}{15} + \frac{2}{15} = ?$

4. $\frac{5}{6} + \frac{1}{6} + \frac{3}{6} = ?$

5. What must you do before you can add 5 ft. and 7 yd.?

6. For the same reason, what must you do before you can add $\frac{5}{7}$, $\frac{3}{4}$, and $\frac{7}{8}$? before you can subtract $\frac{7}{10}$ from $\frac{3}{4}$?

7. Are $\frac{5}{6}$ and $\frac{1}{6}$ *like-fractions*? Are $\frac{5}{6}$ and $\frac{5}{8}$? Like-fractions have a common $__$.

We can count, add, or subtract like-fractions just as we count, add, or subtract whole numbers.

8. To add $\frac{2}{3} + \frac{3}{4} + \frac{1}{2}$, why do we change the fractions to $\frac{8}{12} + \frac{9}{12} + \frac{6}{12}$? Explain how the changes were made.

9. What is the **common denominator** of $\frac{5}{8}$ and $\frac{3}{4}$?

10. In adding $\frac{5}{6} + \frac{3}{4}$, why is 24 a common denominator? Is 24 the smallest common denominator of $\frac{5}{6}$ and $\frac{3}{4}$? Why do we usually use the smallest common denominator?

To find the smallest common denominator of several fractions, first look at the denominators of the given fractions to see if one of them is a common denominator. If it is not, multiply the largest denominator by 2, then by 3, then by 4, and so on, until a common denominator is found. The first one you find this way will be the **lowest common denominator** (L.C.D.)

[W]

Study the work in boxes A–D until you understand how each answer is obtained. Then copy and work Ex. 11–18. Be sure that all your answers are in best form.

A $\begin{array}{r} \frac{7}{8} \\ + \frac{3}{8} \\ \hline \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4} \end{array}$	B $\begin{array}{r} 2\frac{7}{10} \\ + 3\frac{1}{10} \\ \hline 5\frac{8}{10} = 5\frac{4}{5} \end{array}$	C $\begin{array}{r} \frac{5}{12} = \frac{5}{12} \\ + \frac{2}{3} = \frac{8}{12} \\ \hline 1\frac{13}{12} = 1\frac{1}{12} \end{array}$	D $\begin{array}{r} 3\frac{4}{5} = 3\frac{12}{15} \\ + 7\frac{1}{3} = 7\frac{5}{15} \\ \hline 10\frac{17}{15} = 11\frac{2}{15} \end{array}$
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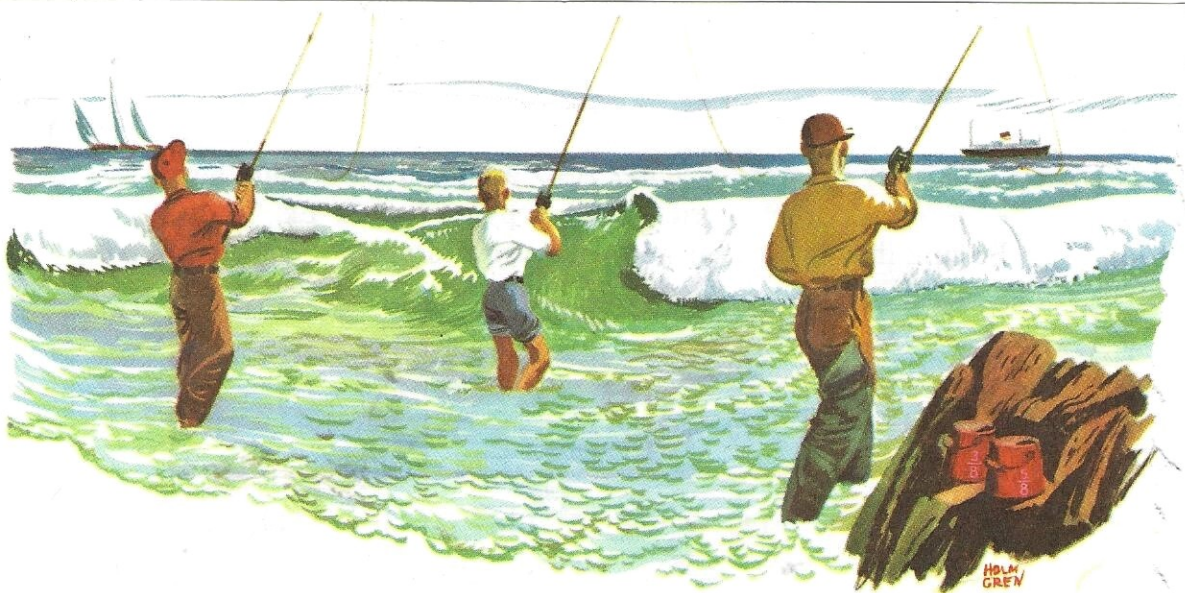
- | | | | |
|---|---|---|---|
| 11. $\begin{array}{r} 3\frac{2}{5} \\ + 6\frac{4}{5} \\ \hline \end{array}$ | 12. $\begin{array}{r} 7\frac{3}{8} \\ + 5\frac{7}{8} \\ \hline \end{array}$ | 13. $\begin{array}{r} \frac{3}{5} \\ + \frac{7}{10} \\ \hline \end{array}$ | 14. $\begin{array}{r} 2\frac{1}{2} \\ + 1\frac{2}{3} \\ \hline \end{array}$ |
| 15. $\begin{array}{r} 2\frac{2}{3} \\ + 7\frac{1}{3} \\ \hline \end{array}$ | 16. $\begin{array}{r} 1\frac{2}{3} \\ + 6\frac{1}{3} \\ \hline \end{array}$ | 17. $\begin{array}{r} 6\frac{2}{3} \\ + 7\frac{5}{6} \\ \hline \end{array}$ | 18. $\begin{array}{r} 15\frac{3}{4} \\ + 17\frac{1}{8} \\ \hline \end{array}$ |

Study boxes E–H. Then copy and work Ex. 19–34.

E $\begin{array}{r} 7\frac{3}{4} \\ - 5\frac{1}{4} \\ \hline 2\frac{2}{4} = 2\frac{1}{2} \end{array}$	F $\begin{array}{r} 2\frac{13}{24} = 2\frac{13}{24} \\ - \frac{1}{2} = \frac{12}{24} \\ \hline 2\frac{1}{24} \end{array}$	G $\begin{array}{r} 3\frac{1}{6} = 2\frac{7}{6} \\ - 1\frac{5}{6} = 1\frac{5}{6} \\ \hline 1\frac{2}{6} = 1\frac{1}{3} \end{array}$	H $\begin{array}{r} 7\frac{1}{2} = 7\frac{8}{16} = 6\frac{24}{16} \\ - 2\frac{11}{16} = 2\frac{11}{16} \\ \hline 4\frac{13}{16} \end{array}$
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- | | | | |
|---|---|---|---|
| 19. $\begin{array}{r} 6\frac{7}{12} \\ - 5\frac{3}{12} \\ \hline \end{array}$ | 20. $\begin{array}{r} 14\frac{1}{4} \\ - 12\frac{3}{4} \\ \hline \end{array}$ | 21. $\begin{array}{r} 3\frac{1}{8} \\ - 1\frac{1}{3} \\ \hline \end{array}$ | 22. $\begin{array}{r} 4\frac{3}{4} \\ - 2\frac{3}{10} \\ \hline \end{array}$ |
| 23. $\begin{array}{r} 13\frac{6}{7} \\ - 8\frac{3}{7} \\ \hline \end{array}$ | 24. $\begin{array}{r} 12\frac{3}{8} \\ - 7\frac{5}{8} \\ \hline \end{array}$ | 25. $\begin{array}{r} 8\frac{7}{10} \\ - 5\frac{2}{5} \\ \hline \end{array}$ | 26. $\begin{array}{r} 23\frac{1}{3} \\ - 19\frac{3}{4} \\ \hline \end{array}$ |
| 27. $\begin{array}{r} 35\frac{7}{8} \\ + 16\frac{3}{4} \\ \hline \end{array}$ | 28. $\begin{array}{r} 45\frac{3}{5} \\ - 37\frac{1}{2} \\ \hline \end{array}$ | 29. $\begin{array}{r} 16\frac{3}{5} \\ + 15\frac{3}{4} \\ \hline \end{array}$ | 30. $\begin{array}{r} 31\frac{1}{3} \\ - 19\frac{1}{2} \\ \hline \end{array}$ |
| 31. $\begin{array}{r} 27\frac{1}{5} \\ - 19\frac{2}{3} \\ \hline \end{array}$ | 32. $\begin{array}{r} 18\frac{7}{8} \\ + 72\frac{3}{4} \\ \hline \end{array}$ | 33. $\begin{array}{r} 45\frac{5}{6} \\ - 19\frac{1}{4} \\ \hline \end{array}$ | 34. $\begin{array}{r} 37\frac{1}{5} \\ - 18\frac{3}{4} \\ \hline \end{array}$ |

► **Extra Practice.** Work Sets 31–40.



The Fishing Trip

Differentiating A. and S. with fractions [W]

Write your work. Give answers in best form.

Jack, Sam and Uncle Tom went surf fishing one day.

1. The boys carried pails of bait with signs " $\frac{3}{8}$ oz." and " $\frac{5}{8}$ oz." to label the sizes of the bait. How much heavier was a piece of $\frac{5}{8}$ -ounce bait than a piece of $\frac{3}{8}$ -ounce bait?

2. Sam used two pieces of bait at one time, one from each pail. How much did the two pieces weigh?

3. In casting their lines, the boys tried for distance. Jack's line once reeled off $95\frac{2}{3}$ ft., and Sam's, $104\frac{1}{2}$ ft. How much more line did Sam put out that time than Jack?

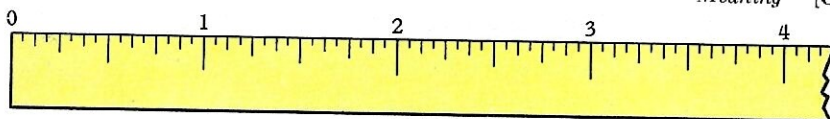
4. During the day, Jack caught a sea bass weighing $6\frac{1}{2}$ lb. and a $10\frac{3}{4}$ -pound flounder. Sam caught only one pollack but it weighed $17\frac{1}{4}$ lb. Which boy caught more fish by weight?

5. Uncle Tom was an old hand at fishing. He caught two sea bass weighing $6\frac{1}{2}$ lb. and $7\frac{1}{4}$ lb., a bluefish weighing $15\frac{1}{2}$ lb., and three pollack which weighed $17\frac{1}{2}$ lb., $15\frac{3}{4}$ lb., and 19 lb. How much fish by weight did Uncle Tom catch?

6. How much more fish by weight did Uncle Tom catch than Jack and Sam caught together?

Multiplying with Fractions

Meaning [O]



Use only the ruler to find the answers for Ex. 1-4.

1. $3\frac{1}{2} \times \frac{1}{2}$ in. = ? Starting at 0, find the multiplicand ($\frac{1}{2}$ in.) on the ruler. Then count to the right as many $\frac{1}{2}$ in. as the multiplier, $3\frac{1}{2}$, tells you to. ($3 \times \frac{1}{2}$ in. = $\frac{3}{2}$ in.; $\frac{1}{2}$ of $\frac{1}{2}$ in. = $\frac{1}{4}$ in.; so $3\frac{1}{2} \times \frac{1}{2}$ in. = $1\frac{3}{4}$ in.)

2. $\frac{1}{3} \times \frac{3}{4}$ in. = ? 3. $2\frac{1}{3} \times 1\frac{1}{2}$ in. 4. $2 \times 1\frac{1}{4}$ in. = ?

Now explain the work in the boxes below.

A $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	B $\frac{3}{4} \times 9 = \frac{3}{4} \times \frac{9}{1} = \frac{27}{4} = 6\frac{3}{4}$
C $\frac{4}{9} \times 1\frac{5}{8} = \frac{4}{9} \times \frac{13}{8} = \frac{13}{18}$	D $3\frac{1}{7} \times 4\frac{1}{5} = \frac{22}{7} \times \frac{21}{5} = \frac{66}{5} = 13\frac{1}{5}$

To multiply a fraction by a fraction, we multiply the denominators to get the size of the new fractional unit and multiply the numerators to get the total number of equal parts.

A short method of multiplying a mixed number and a whole number is shown at the right. Explain the steps.

Multiply. Use short methods whenever you can. [W]

$$\begin{array}{r} 16 \\ \times 2\frac{3}{4} \\ \hline 12 \text{ } (\frac{3}{4} \text{ of } 16) \\ 32 \text{ } (2 \times 16) \\ \hline 44 \end{array}$$

5. $\frac{2}{3} \times \frac{9}{10}$ 6. $1\frac{3}{5} \times 2\frac{1}{4}$ 7. $\frac{3}{5} \times \frac{1}{9}$ 8. $5\frac{1}{4} \times \frac{6}{7}$
 9. $3\frac{1}{5} \times 2\frac{3}{4}$ 10. $4\frac{2}{3} \times 9$ 11. $8 \times 5\frac{3}{4}$ 12. $2\frac{1}{2} \times 12$

➤ **Extra Practice.** Work Sets 41-45.

Dividing with Fractions

Meaning; common-denominator method [O]

First Method. Dividing fractions is not difficult if you remember that in a fraction the numerator always tells how many and the denominator shows the kind of equal parts. In the fraction $\frac{4}{5}$, the numerator shows that there are 4 equal parts, and the denominator shows that the size of the fractional unit is $\frac{1}{5}$.

Working with whole numbers or with fractions, we think much the same way:

$$4 \text{ apples} \div 2 \text{ apples} = 4 \div 2, \text{ or } 2$$

$$4 \text{ fifths} \div 2 \text{ fifths} = 4 \div 2, \text{ or } 2$$

$$\frac{4}{5} \div \frac{2}{5} = 4 \div 2, \text{ or } 2$$

Explain the work in boxes A-D.

A

$$\frac{3}{4} \div \frac{1}{6} = \frac{9}{12} \div \frac{2}{12} = 9 \div 2 = 4\frac{1}{2}$$

B

$$6 \div \frac{5}{7} = \frac{42}{7} \div \frac{5}{7} = 42 \div 5 = 8\frac{2}{5}$$

C

$$1\frac{3}{8} \div 2 = \frac{11}{8} \div \frac{16}{8} = 11 \div 16 = \frac{11}{16}$$

D

$$3\frac{3}{5} \div 1\frac{1}{3} = \frac{18}{5} \div \frac{4}{3} = \frac{54}{15} \div \frac{20}{15} = 54 \div 20 = 2\frac{14}{20} = 2\frac{7}{10}$$

To divide with fractions, you can change both dividend and divisor to like-fractions. Then divide, using the numerators of the like-fractions.

[W]

Divide, using the **common-denominator method**.

1. $2\frac{3}{4} \div 2$

4. $\frac{15}{16} \div \frac{5}{12}$

7. $4\frac{1}{5} \div 3$

10. $5\frac{1}{2} \div 2\frac{1}{3}$

2. $\frac{5}{7} \div 3$

5. $4\frac{3}{5} \div 1\frac{1}{2}$

8. $7 \div 3\frac{1}{7}$

11. $\frac{5}{9} \div \frac{2}{3}$

3. $1\frac{1}{2} \div \frac{1}{4}$

6. $6\frac{2}{3} \div \frac{2}{3}$

9. $3\frac{1}{4} \div \frac{1}{5}$

12. $8 \div \frac{5}{6}$

Second Method (a short cut). $4 \div 3 = ?$

$4 \div 3$ can be written in any of these ways because they all mean the same: $\frac{4}{1} \div \frac{3}{1}$; $\frac{1}{3}$ of 4; $\frac{1}{3} \times 4$; $4 \times \frac{1}{3}$. Explain.

Since $\frac{4}{1} \div \frac{3}{1} = 4 \times \frac{1}{3}$, we can find the answer for $\frac{4}{1} \div \frac{3}{1}$ by changing the divisor $\frac{3}{1}$ to $\frac{1}{3}$ and then multiplying by the new fraction. When we change $\frac{3}{1}$ to $\frac{1}{3}$, we are **inverting the divisor**.

Explain the work in boxes E-H below. Each time, tell why it is easier to invert the divisor and multiply than to use the common-denominator method.

E $\frac{7}{8} \div 3 = \frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$	F $9 \div \frac{4}{5} = \frac{9}{1} \times \frac{5}{4} = \frac{45}{4} = 11\frac{1}{4}$
G $4\frac{2}{3} \div \frac{5}{6} = \frac{14}{3} \div \frac{5}{6} = \frac{14}{3} \times \frac{6}{5} = \frac{84}{15} = \frac{28}{5} = 5\frac{3}{5}$	
H $2\frac{1}{2} \div 5\frac{5}{8} = \frac{5}{2} \div \frac{45}{8} = \frac{5}{2} \times \frac{8}{45} = \frac{40}{90} = \frac{4}{9}$	

13. Show on the board how you could have used cancellation to shorten the work in box G; in box H.

Here is a rule for dividing with fractions the short way:

In dividing with fractions, you can change whole numbers and mixed numbers to fractions. Then invert the divisor, cancel when you can, and multiply.

[W]

Divide, using the **inversion method**.

14. $7\frac{1}{2} \div 5$ 17. $6\frac{3}{5} \div \frac{3}{4}$ 20. $17\frac{1}{3} \div 2\frac{1}{6}$ 23. $12\frac{1}{2} \div 8$
 15. $\frac{4}{5} \div 3$ 18. $4\frac{1}{2} \div 6$ 21. $3\frac{2}{3} \div \frac{5}{8}$ 24. $\frac{11}{16} \div \frac{3}{4}$
 16. $\frac{5}{8} \div \frac{2}{3}$ 19. $15 \div \frac{2}{3}$ 22. $\frac{7}{8} \div 1\frac{2}{5}$ 25. $14 \div 1\frac{1}{6}$

➤ **Extra Practice.** Work Sets 46–50.



Ways of Comparing Numbers

Relation of two numbers: ratio [O]

1. Bob had 48 marbles and Henry had 12. Study the four ways given below for comparing the number of marbles Bob had and the number Henry had. Tell how to finish each way.

a. One way to compare would be to find **how many more** marbles Bob had than Henry had. $48 - 12 = ?$

b. Another way would be to find **how many fewer** marbles Henry had than Bob. $48 - 12 = ?$

c. A third way would be to find what fractional part Henry's marbles were of Bob's marbles. 12 marbles is **what part** of 48 marbles? $\frac{12}{48} = \frac{?}{4}$ Henry had $?\frac{?}{?}$ as many as Bob.

d. A fourth way would be to find how many times as many marbles Bob had as Henry had. 48 marbles is **how many times** 12 marbles? $\frac{48}{12} = \frac{?}{1}$, or $?\frac{?}{?}$. Bob had $?\frac{?}{?}$ times as many marbles as Henry.

2. In Ex. 1, a and b, what process do we use in comparing the numbers? What does each answer tell?

3. In c and d, what process do we use in comparing the numbers? What does each answer tell?

For Ex. 4-11, tell each time whether the answer is a what-part-of number or a how-many-times number.

4. $\frac{36}{6} = \frac{6}{1}$

5. $\frac{35}{56} = \frac{5}{8}$

6. $\frac{14}{21} = \frac{2}{3}$

7. $\frac{15}{3} = \frac{5}{1}$

8. $\frac{6}{8} = \frac{3}{4}$

9. $\frac{12}{9} = \frac{4}{3}$

10. $\frac{15}{25} = \frac{3}{5}$

11. $\frac{24}{8} = \frac{3}{1}$

[O]

When we compare two numbers by division, we say that we find the **ratio** (the relationship) of one number to another. We find either *how many times* as large one number or amount is as another or *what part* one number or amount is of another. See the boxes below.

<i>How many times?</i>	<i>What part of?</i>
Bob's marbles, 48 $\div$ Henry's marbles, 12 = $\frac{4}{1}$, or 4	Henry's marbles, 12 $\div$ Bob's marbles, 48 = $\frac{1}{4}$

We often write the terms of a ratio in fraction form, but we can use a colon between the numbers. The ratio of 12 to 48 is $\frac{12}{48}$ or 12 : 48. Either form may be read, "12 to 48."

A ratio, like a fraction, is generally reduced to best form. Thus the ratio of 4 to 12 is $\frac{1}{3}$ or 1 : 3; and the ratio of 12 to 4 is $\frac{3}{1}$ or 3 : 1 or 3.

12. Dick is 48 in. tall and Joe is 5 ft. tall. Is the ratio of Dick's height to Joe's height $\frac{48}{5}$? This does not make sense, because it means that Dick is almost 10 times as tall as Joe. We should have expressed both heights in the same unit of measure before we compared them.

48 in. = 4 ft., so the ratio of Dick's height to Joe's is $\frac{4}{5}$.

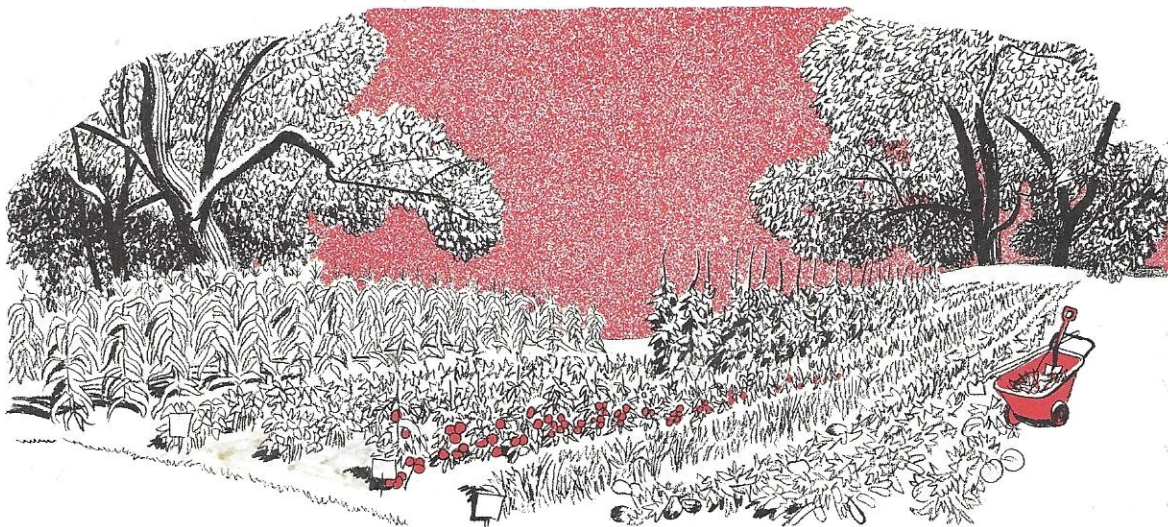
13. Change Joe's height to inches and write on the board the ratio comparing his height with Dick's.

Two numbers to be compared must be expressed in the same unit of measure.

[W]

In Ex. 14-22, give for each pair of numbers both the what-part-of ratio and the ratio that shows how many times. Write each ratio first as a fraction and then with the colon.

- | | | |
|----------------------|---------------------|---------------------|
| 14. 9 ft.; 12 ft. | 17. 144 in.; 12 in. | 20. 34 ft.; 17 ft. |
| 15. 25 mi.; 125 mi. | 18. 2 lb.; 4 oz. | 21. 9 qt.; 21 qt. |
| 16. 12 min.; 36 sec. | 19. 52 bu.; 13 bu. | 22. 95 lb.; 100 lb. |



Ratios in the Garden

Whole numbers [W]

Write your work for these problems:

1. In a garden 50 ft. long and 25 ft. wide, what is the ratio of the width to the length? of the length to the width?
2. To control leaf-eating insects, Mr. Bates used a dust made of one part arsenate of lead and 5 parts lime. What was the ratio of arsenate of lead to lime?
3. Of 100 flagstones which Mr. Bates bought for the path leading to his garden, 10 were red, 35 gray, 40 green, and 15 black. Find the ratio of **a.** red to green; **b.** gray to black.
4. Mr. Bates planted 2 lb. of beans and harvested 70 lb. What is the ratio of pounds harvested to pounds planted?
5. When a gardener speaks of a 10-6-4 fertilizer, he means that out of every 100 parts, 10 parts are nitrogen, 6 parts are phosphorus, and 4 parts are potash. The rest is material which holds these fertilizing elements together. Give the ratio of **a.** nitrogen to potash; **b.** phosphorus to potash; **c.** all fertilizing elements to other material.
6. Instructions for storing home-grown celery specify that the trench should be dug as deep as the plants are tall. What is the ratio between height of plants and depth of trench?

What Is the Ratio?

Fractions; how many times or what part of [O]

As with whole numbers, we find the relationship (ratio) between two fractions by comparing them through division. Some quotients tell *how many times*, and some, *what part of*.

1. Will a ratio that shows how many times ever be less than 1?

2. Explain for each fraction example in box A why the quotient will be a ratio that shows how many times.

3. In box A, which of the numbers in each example is the dividend?

4. When the ratio shows how many times, which number is larger—the dividend or the divisor? (Study the examples in box A.)

5. In box B, tell why each quotient will be a what-part-of ratio.

6. In box B which number in each example is the dividend?

7. In a what-part-of ratio, which number is larger—the dividend or the divisor? (Study the examples in box B.)

8. Work each example in box A. If any quotient is not a how-many-times number, it is wrong. Do that example again.

9. Work each example in box B. If any quotient is not a what-part-of number, it is wrong. Do that example again.

A

$$8 \div \frac{2}{3} = ?$$

$$6 \div 1\frac{1}{2} = ?$$

$$\frac{3}{4} \div \frac{1}{2} = ?$$

$$2\frac{1}{2} \div \frac{5}{8} = ?$$

$$4\frac{2}{3} \div 3\frac{1}{2} = ?$$

$$3\frac{1}{3} \div 2 = ?$$

B

$$\frac{2}{3} \div 6 = ?$$

$$\frac{1}{2} \div \frac{3}{4} = ?$$

$$\frac{3}{8} \div 1\frac{3}{4} = ?$$

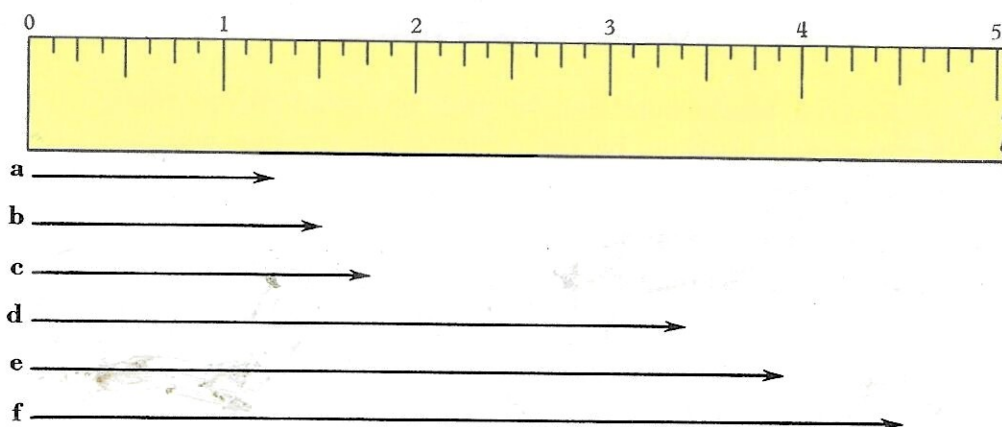
$$2\frac{1}{2} \div 5 = ?$$

$$5 \div 7\frac{1}{2} = ?$$

$$3\frac{3}{4} \div 4\frac{3}{8} = ?$$

If the dividend is larger than the divisor, the quotient is more than 1 and it tells how many times as large as the divisor the dividend is.

If the dividend is smaller than the divisor, the quotient is less than 1 and it tells what part of the divisor the dividend is.



Rounding and Estimating with Fractions

Meaning [O]

1. Is line a more nearly 1 inch or 2 inches long? Rounding $1\frac{1}{4}$ inches to the nearest inch, we can think that the fraction $\frac{1}{4}$ is less than $\frac{1}{2}$, so the length of line a is about $_\ ? _\$ inch.

2. Is line c more nearly 1 inch or 2 inches long? Because the fraction $\frac{3}{4}$ is more than $\frac{1}{2}$, we round $1\frac{3}{4}$ inches to 2 inches.

3. Because line b is exactly $1\frac{1}{2}$ inches long, we round it to the next larger inch, or to $_\ ? _\$ inches.

4. Tell to the nearest whole number of inches the length of line d; line e; line f.

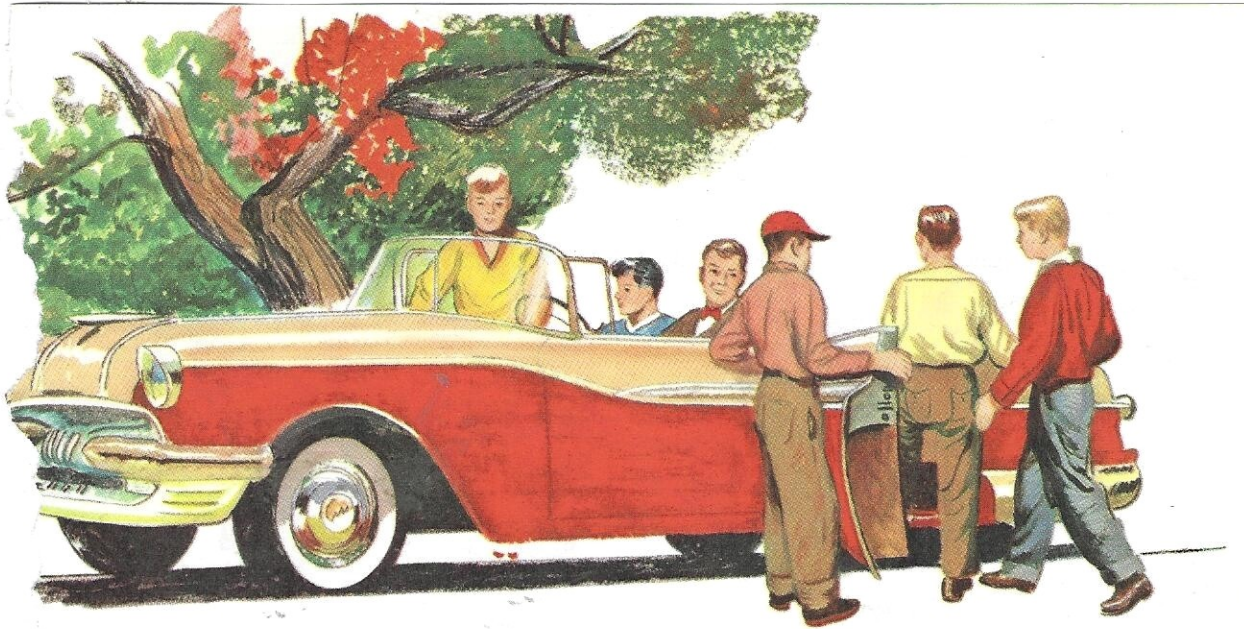
In rounding to the nearest whole number: (a) when a fraction is equal to or greater than $\frac{1}{2}$, add 1 to the whole number and drop the fraction; (b) when the fraction is less than $\frac{1}{2}$, drop the fraction.

In rounding, to help you know whether any fraction is $\frac{1}{2}$ or less than or more than $\frac{1}{2}$, remember these things:

a. In fractions like $\frac{3}{6}$ and $\frac{12}{24}$, the numerator is just half the denominator, so such fractions equal $\frac{1}{2}$.

b. In fractions like $\frac{7}{8}$ and $\frac{4}{5}$, the numerator is greater than one half the denominator, so such fractions equal more than $\frac{1}{2}$.

c. In fractions like $\frac{2}{7}$ and $\frac{4}{15}$, the numerator is less than one half the denominator, so such fractions equal less than $\frac{1}{2}$.



Off for the Game!

Problems with fractions; estimating [W]

Before working each problem, write an approximate answer.

Paul's brother Jim said that he would take 5 boys to the football game at Bixton if they wanted to share expenses.

1. Just as they started for the game Jim bought $8\frac{4}{5}$ gal. of gasoline at 32¢ a gallon. How much did he pay the attendant?

2. On the way, each boy had 2 hamburgers and a glass of milk. Jim paid the bill, which came to \$3.24. What was each one's share of this bill—that is, what was $\frac{1}{6}$ of the cost?

3. Jim averaged 48 miles an hour during the trip down, which took $3\frac{1}{2}$ hr. How far did the boys drive to the game?

HINT. In estimating a product, when one factor contains $\frac{1}{2}$, round only the other factor.

4. Gasoline for the return trip cost $1\frac{1}{2}$ ¢ less a gallon than at home. Jim bought $9\frac{1}{2}$ gal. this time. How much less did he pay than he would have paid at home?

HINT. In estimating a product, when both factors contain $\frac{1}{2}$, round only the larger factor.

5. Find each boy's share of the cost,— $\frac{1}{6}$ of \$8.96.

Practice with Fractions

Work the first row of each set of exercises below. If you make mistakes in any set, then work the other rows of that set. [W]

Add.

a	b	c	d	e
1. $\frac{5}{8}$	$2\frac{3}{5}$	$3\frac{1}{2}$	$6\frac{3}{5}$	$14\frac{2}{3}$
$+\frac{7}{8}$	$+\frac{4}{5}$	$+5\frac{1}{4}$	$+2\frac{3}{10}$	$+29\frac{3}{4}$
2. $5\frac{3}{4}$	$1\frac{5}{9}$	$5\frac{1}{6}$	$7\frac{5}{8}$	$47\frac{1}{3}$
$+2\frac{1}{4}$	$+3\frac{7}{9}$	$+3\frac{1}{2}$	$+2\frac{3}{4}$	$+38\frac{1}{2}$
3. $3\frac{11}{12}$	$6\frac{3}{10}$	$4\frac{1}{5}$	$6\frac{5}{6}$	$75\frac{2}{3}$
$+4\frac{3}{12}$	$+5\frac{8}{10}$	$+2\frac{1}{2}$	$+2\frac{2}{3}$	$+89\frac{7}{8}$

Subtract.

4. $\frac{7}{8}$	$4\frac{5}{8}$	$3\frac{5}{8}$	$\frac{3}{9}$	$56\frac{3}{4}$
$-\frac{3}{8}$	$-\frac{7}{8}$	$-1\frac{1}{4}$	$-\frac{1}{3}$	$-29\frac{1}{3}$
5. $7\frac{3}{10}$	$3\frac{3}{5}$	$6\frac{2}{3}$	$5\frac{7}{10}$	$25\frac{1}{5}$
$-2\frac{7}{10}$	$-2\frac{4}{5}$	$-4\frac{1}{6}$	$-3\frac{1}{2}$	$-17\frac{1}{2}$

Multiply.

6. $5 \times \frac{7}{8}$	$6 \times 3\frac{3}{5}$	$\frac{3}{5} \times \frac{7}{8}$	$21 \times 4\frac{1}{5}$	$\frac{5}{8} \times \frac{3}{5}$
7. $3 \times 4\frac{2}{3}$	$1\frac{1}{2} \times 5\frac{1}{2}$	$2\frac{1}{3} \times 4\frac{1}{2}$	$3\frac{1}{5} \times 7$	$2\frac{3}{8} \times \frac{4}{7}$
8. $\frac{1}{2} \times 7$	$\frac{7}{8} \times 1\frac{1}{8}$	$13 \times 7\frac{1}{8}$	$4\frac{3}{8} \times 2\frac{1}{3}$	$1\frac{1}{2} \times \frac{8}{9}$

Divide.

9. $\frac{3}{4} \div 4$	$\frac{3}{5} \div 3$	$11 \div 3\frac{1}{4}$	$7\frac{1}{2} \div \frac{1}{2}$	$\frac{9}{10} \div \frac{3}{5}$
10. $\frac{3}{8} \div 10$	$\frac{4}{5} \div \frac{1}{5}$	$2\frac{1}{2} \div 1\frac{1}{4}$	$2\frac{1}{4} \div 4\frac{1}{2}$	$7\frac{1}{2} \div 5$
11. $10 \div \frac{1}{10}$	$\frac{1}{10} \div 10$	$1 \div \frac{1}{10}$	$\frac{1}{10} \div 1$	$2\frac{2}{3} \div 4$

► Extra Practice. Work Set 51.

Arithmetic with Words and with Numbers

1-, 2-, and 3-step problems [W]

Often you are asked to work *word problems*. At other times you have *examples* with signs. The actual computation that you do may be the same in a problem and in an example.

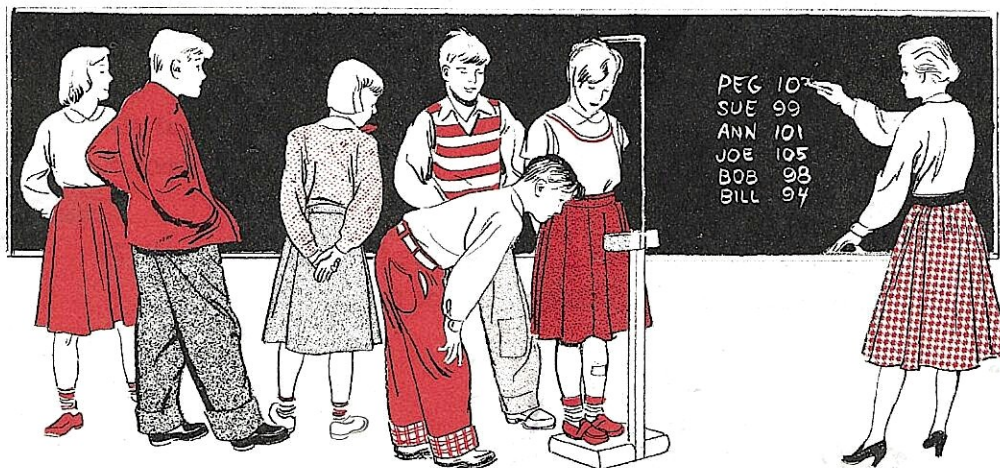
Study the boxes. Then copy and finish the examples.

a		b
1.	Joe weighs 43 lb. more than his young brother Eddie. How much does Eddie weigh?	$\begin{array}{r} 105 \\ - 43 \\ \hline ? \end{array}$
2.	The picture below shows the weights of six children. What is the average weight of the boys?	$\begin{array}{r} 105 \quad ? \\ 98 \quad 3 \overline{)297} \\ + 94 \\ \hline 297 \end{array}$
3.	From the picture find how much more Peg weighs than the average weight of the girls.	$\begin{array}{r} 101 \quad ? \\ 99 \quad 3 \overline{)303} \\ + 103 \\ \hline 303 \\ 103 - 101 = ? \end{array}$

[O]

4. To solve problem 1, you subtract. For problem 2, you add and $\div$. For problem 3, you $\div$, $\div$, and $\div$.

Problems like Ex. 1 are *one-step* problems, those like Ex. 2 are *two-step*, and those like Ex. 3 are *three-step*.



Practice with Problems

1-, 2-, 3-step problems [O]

1. How many steps are there in a problem which asks how much longer one movie is than another? You know the time each movie begins and ends.

2. Is a two-step problem more difficult than a one-step problem or is it only longer? Explain.

3. Can a three-step problem be broken up into three one-step problems? Does it make any difference which step is worked first? Explain.

Making problems and supplying numbers [W]

4. Make a problem about the heights of five different trees and their average height. Supply your own numbers.

a. How many steps are there in a problem like this?

b. Work your problem.

5. a. If you know the price per pound of butter and the number of pounds bought, how many steps will be needed to find the total cost?

b. Supply some appropriate numbers and make a problem to fit part a.

c. Solve the problem you have made.

Solving problems, numbers supplied

For each of Ex. 6–9, first write on your paper the number of steps in the problem. Then work the problem.

6. Mr. Jones bought 12 gal. of gasoline at 32¢ per gallon. How much was his bill?

7. Mr. Jones drove 108 mi. If his car travels 18 mi. on 1 gal. of gasoline, what will be the cost of gasoline for this trip at 32¢ per gallon?

8. There were 684 people at the first show and 596 at the second. If tickets were 50¢ each, find the average amount taken in per show.

9. Find how much John Shea makes for himself on a \$275 sale if he receives for himself $\frac{1}{25}$ of the sale price.

Understanding the Problem

Reading carefully [O]

When you start to solve a problem, the first thing to do is to read the problem carefully. Make sure that you understand just what facts are given and what you are asked to find. Then plan a way for finding the answer.

Problems are not always stated in the simplest way. If you are not wide awake you may miss some of the given facts or you may not notice that you have a two- or a three-step problem.

Read problems 1-3 carefully and tell what facts are given.

1. Alex rode 4 mi. on his bicycle in 12 min. What was his speed in miles per hour?

2. Dick bought 3 sodas at 15¢ each. How much change should he receive from \$1?

3. Mr. Black's car travels an average of 15 mi. per gallon of gasoline. At 32¢ per gallon, how much will the gasoline cost for a 450-mile trip?

How many steps are needed in solving each of problems 1-3? Tell what you would do in solving each problem.

Now read problems 4 and 5 carefully and tell what you are asked to find and what you would do to solve each problem.

4. Last week Dick worked 13 hr. at a wage of 65¢ per hour. How much did he earn?

5. Mr. Wood bought 12 lb. of grass seed at 75¢ per pound and 3 boxes of pansies at 45¢ per box. What was his bill?

Solving the problems [W]

Now work problems 1-5.



Deciding Which Number Process to Use

A., S., M., D. [O]

In solving problems you need to decide when to add, subtract, multiply, or divide. You decide by thinking about the meaning of the problem. Study the following:

A. Mrs. Black bought \$2.35 worth of meat and \$1.20 worth of fruit. How would you find her total bill?

B. Mrs. White bought 7 lb. of meat at 65¢ per pound. How would you find her total bill?

1. Why do you add for Ex. A but multiply for Ex. B?
2. Could you find the answer to Ex. B by adding? Explain.

C. At a sale, Mr. Black paid \$5.18 for two shirts. One of them cost \$2.29. How would you find the cost of the other?

D. Mr. Brown bought \$3.75 worth of oranges that were priced at 75¢ per dozen. How would you find how many dozen he bought?

3. Why do you subtract in Ex. C but divide in Ex. D?
4. Could you find the answer to Ex. D by subtracting?
5. What general rules can you give to help you decide when to add, subtract, multiply, or divide?

[W]

Before solving problems 6–9, think carefully what processes you should use. Then work the problems.

6. One fifth of Mrs. Sampson's 245 chickens were white and the rest were brown. How many were white?

7. Mr. Black invested \$7,825 in bonds and \$5,675 in real estate. What was his total investment?

8. Dick received \$28.96 for selling papers over a period of 4 weeks. How much did he average a week?

9. An airplane covered 753 mi. in 3 hr. What was its average speed in miles per hour?

Does the Answer Make Sense?

Remainder in D. [O]

1. Ted works in the public library on Saturdays. One day he had to carry a big stack of books downstairs. There were 122 books in the pile and he could carry 14 at a time. How many trips did he make?

Joan said that the answer was $8\frac{5}{7}$ but Nancy said it was 9. Which answer made sense?

Joan's	Nancy's
$\begin{array}{r} 8\frac{5}{7} \\ 14 \overline{)122} \\ \underline{112} \\ 10 \end{array}$	$\begin{array}{r} 8, R10 \\ 14 \overline{)122} \\ \underline{112} \\ 10 \end{array}$
$\frac{10}{14} = \frac{5}{7}$	

The remainder in division may be shown in a fraction or with R. To make the answer sensible, the quotient is sometimes rounded to the next larger whole number or to the next smaller whole number. Each time, study the problem to decide which way is best.

Which way should the answer be given in Ex. 2-4?

2. Divide 6 yd. of ribbon into 4 equal parts.
3. Find the number of truck loads needed to transport 23 cattle, 4 at a time.
4. Share 53¢ among 3 boys.

[W]

Work the following, giving sensible answers.

5. Al has a pole 12 ft. long. He wants to cut it into 15-inch pieces. How many 15-inch pieces can he make?
6. Mr. Webb's car went 134 mi. on 8 gal. of gasoline. How many miles did it average to a gallon?
7. Don is putting potatoes into 5-pound bags. If there are 322 lb. of potatoes in all, how many bags can he fill?
8. Miss Harris brought 2 doz. chocolate bars to the picnic, but only 18 people came. She divided the chocolate bars equally among the 18 people. What was each person's share?



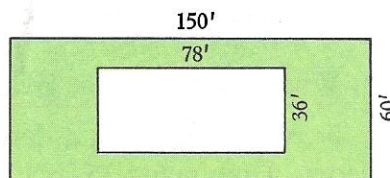
The Smith Family's Playground

Data extra or missing [W]

Sometimes you may have a problem with extra numbers, that is, numbers that are not needed for the solution. Before solving, decide which numbers are needed. For example, in problem 1, do you need to use the number 1955? Watch out for extra numbers as you work problems 1-6.

1. In 1955, Tom Smith's father built a new house on a lot 150 ft. across the front and 260 ft. from front to back. He planned to use the last 60 ft. at the back of the lot for a play area. Centered in this strip, he made the tennis court as shown at the right.

A ball could fall ? ft. beyond one end of the court and be on Mr. Smith's land.



2. Tom earned \$95 during vacation. One day he bought two tennis balls for 65¢ each, and a net for \$12.35. If he paid for these with a \$20 bill, how much change did he get?

3. Tom bought 500 ft. of cloth tape to mark off the court. At one store the price was \$16, with a discount of \$1.60; at another store, it was \$15.25. At which store, the first or the second, was the tape cheaper? How much cheaper?

4. Ann Smith is 6 yr. younger than Tom. From June 1 to August 15, she won 58 sets of tennis and lost 17. What is the ratio of the sets lost to sets played?

5. Tom could buy balls for 65¢ each or \$1.85 for 3 or \$7.25 a dozen. Find the cost per ball, bought 3 at a time.

6. Ann had a racket which had cost \$7.50. Then she bought for \$4 a used racket that had originally cost \$6.50. How much less than the original cost did she pay?

[O]

Sometimes you cannot solve a problem because not enough numbers are given. In Ex. 7-11, tell

(1) what else you need to know in order to solve;

(2) what numbers are not needed.

7. In a store window at 825 Fourth Street, Tom saw 3 rackets with original price tags as follows: \$9.50, \$8.25, \$7.50. There was also a sign, "All rackets now at a fraction of the regular price!" How much would each of the 3 rackets cost?

8. How many shrubs would be needed for a hedge 3 ft. high between the play area and the other part of the lot?

9. Mrs. Smith wanted enough benches for the play area so that 9 people could sit down at one time. The benches cost \$4 each. What would be the total cost of the benches?

10. To make a seesaw for the 6-year-old twins, their Uncle Tom trimmed 3 in. off each end of a plank 10 in. wide and 2 in. thick. How long was the seesaw plank?

11. The twins wanted a rope swing. For the seat, Uncle Tom bored $\frac{3}{4}$ -inch holes 16 in. apart in a piece of board 8 in. wide. He ran rope through the holes, allowing 3 ft. at each end for fastening the rope to a branch. About how much rope did Uncle Tom use?

[W]

For Ex. 7-11, supply reasonable numbers for the numbers that are missing. Then solve the problems.

What Is Missing?

$$n \times 3 = 6$$

$$2 \times n = 6$$

$$2 \times 3 = n$$

$$6 = 2 \times n$$

$$6 = n \times 3$$

$$n = 2 \times 3$$

Finding n , the Missing Number

Fundamental idea for 3 types of problems; M. and D. [O]

If you understand **the relationship between a product and its two factors**, it will help you to work with problems.

1–6. Tell how you would find n for each of the examples in the box, above.

7. In which examples in the box do you find a product? a missing factor?

Tell how to find n in each of the following:

8. $72 = 8 \times n$ 9. $n \times 15 = 90$ 10. $845 = n \times 169$
11. $n = 14 \times 8$ 12. $n \times 17 = 51$ 13. $98 = 14 \times n$

To find a missing product, multiply its factors together.

To find a missing factor, divide the product by the other factor.

[W]

Find n , the missing number, in the following:

- | a | b | c |
|------------------------|-----------------------|-----------------------|
| 14. $16 = n \times 2$ | $n = 25 \times 64$ | $38 \times 59 = n$ |
| 15. $n = 71 \times 56$ | $483 = 23 \times n$ | $n = 37 \times 65$ |
| 16. $98 = 7 \times n$ | $46 \times 75 = n$ | $44 \times n = 2,332$ |
| 17. $49 = n \times 7$ | $25 \times n = 925$ | $54 \times 29 = n$ |
| 18. $n = 19 \times 28$ | $n \times 35 = 1,365$ | $n \times 52 = 4,056$ |
| 19. $n = 56 \times 83$ | $n = 89 \times 72$ | $n = 63 \times 91$ |
| 20. $78 \times 39 = n$ | $28 \times 48 = n$ | $51 \times n = 4,692$ |

Factors and Their Product Are Used in Problems

Notice in the box below how three different problems can be made from the same set of numbers. Finish the work at the board.

I. Mr. Walker sold 13 radios at \$49 each. How much did he receive?	(Two factors are given. You multiply to find the product.) $13 \times \$49 = n$
II. At \$49 each, Mr. Walker received \$637 for some radios. How many radios did he sell?	(The product and one factor are given. You divide to find the other factor.) $n \times \$49 = \637 ; $n = \$637 \div \49 , or $_{-?}$.
III. Mr. Walker received in all \$637 for 13 radios. How much did he get per radio?	(The product and one factor are given. You divide to find the other factor.) $13 \times n = \$637$; $n = \$637 \div 13$, or $_{-?}$.

For each of the following problems, write the work as in the right-hand boxes above. Then finish the work.

1. Dick bought 6 books for \$2 each. How much did he spend for the books?

2. Mr. Rand received \$630 for 210 bu. of apples. How much did he receive per bushel?

3. Mr. Lee sold his farm for \$1,500, receiving a price of \$75 per acre. How many acres were there in the farm?

Now make up two other problems to go with each of problems 1-3.

You have been working with the three types of problems in which you find a missing product or factor.



Three Types of Problems with Fractions

M. and D. relationships [O]

Study I–III below and then answer questions 1 to 3 to see how the product-factor relationship works with fractions.

I. Jim struck out $\frac{1}{8}$ of his 48 times at bat one month. How many times did he strike out? $\frac{1}{8} \times 48 = n$; $n = ?$

II. Jim struck out 6 of his 48 times at bat. What fraction of his times was this? $n \times 48 = 6$; $n = 6 \div 48$, or $\frac{6}{48}$, or $?\dots$

III. Jim struck out 6 times, which was $\frac{1}{8}$ of his times at bat one month. How many times at bat did he have?

$$\frac{1}{8} \times n = 6; \quad n = 6 \div \frac{1}{8}, \quad \text{or} \quad 6 \times \frac{8}{1}, \quad \text{or} \quad ?\dots$$

1. In I, two factors are given and you are asked to find the product. What is given in II? in III?

2. What process do you use in solving I? II? III?

3. What do you notice about the size of the answer in I? in II? in III? Explain.

4–6. Write a type I problem with a fraction factor. ^[W] Then change it to make a type II problem; then a type III problem.

In type I problems, both factors are given and you multiply to find a product.

In type II problems, the product and a factor are given. You divide to find the factor showing a relationship, or ratio.

In type III problems, the product and the ratio factor are given. You divide to find the other factor.

Do You Understand?

Test of Information and Meaning 2

1. Three answers, (a), (b), and (c), are given for the problem below. Which answer is not reasonable, which is an exact answer, and which is an approximate answer?

The auditorium contained 30 rows of seats with 28 seats in each row. How many seats were there altogether?

Answers: (a) 840; (b) 613; (c) 900.

2. Which of the pairs of fractions below are like-fractions?

- (a) $\frac{2}{3}, \frac{1}{3}$ (b) $\frac{3}{4}, \frac{6}{8}$ (c) $\frac{3}{5}, \frac{3}{7}$ (d) $\frac{5}{6}, \frac{7}{6}$

3. If a whole number is divided by a proper fraction, will the quotient be larger or smaller than the dividend? Write and work an example to illustrate your answer.

4. Dick had 13 red marbles and 26 white ones. Which of the following expresses the ratio of red to white marbles?

- (a) 26:13 (b) $\frac{1}{2}$ (c) $\frac{26}{13}$ (d) 2:1

5. Which of the following fractions has the largest value?

- (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{2}{7}$ (d) $\frac{2}{9}$

6. If both the numerator and the denominator of a fraction are divided by the same number, will the value of the fraction be larger, the same, or smaller? Write and work an example to illustrate your answer. Make a drawing.

7. Which of the following are pairs of equivalent fractions?

- (a) $\frac{3}{4}, \frac{9}{12}$ (b) $\frac{2}{6}, \frac{1}{8}$ (c) $\frac{2}{3}, \frac{6}{9}$ (d) $\frac{3}{5}, \frac{15}{25}$ (e) $\frac{4}{8}, \frac{12}{18}$

8. Arrange the following numbers in order of size, with the largest at the left:

- $\frac{4}{5}$ $\frac{6}{10}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{10}$ $\frac{3}{5}$ $\frac{3}{4}$

9. Which of the following fractions are in lowest terms?

- $\frac{6}{14}$ $\frac{3}{8}$ $\frac{1}{5}$ $\frac{6}{8}$ $\frac{9}{10}$ $\frac{14}{16}$ $\frac{1}{25}$

10. If a proper fraction is multiplied by a whole number, will the product be larger than, the same as, or smaller than the whole number?

Do You Make Mistakes?

Diagnostic Test 2

- | | | | | |
|---|----------------------------------|-----------------------------------|-------------------------------------|------------------------------------|
| a | b | c | d | e |
| 1. $\frac{9}{16} + \frac{5}{8} + \frac{3}{4}$ | $\frac{3}{4} + \frac{7}{12}$ | $1\frac{1}{2} + 2\frac{2}{3}$ | $\frac{4}{5} + \frac{7}{10} + 1$ | $6\frac{2}{3} + 9\frac{3}{4}$ |
| 2. $1\frac{5}{8} - 1\frac{5}{16}$ | $6 - \frac{4}{5}$ | $1\frac{5}{6} - \frac{7}{8}$ | $8\frac{1}{2} - 5\frac{3}{5}$ | $2\frac{7}{10} - \frac{1}{2}$ |
| 3. $2\frac{4}{5} \times \frac{3}{8}$ | $\frac{3}{7} \times \frac{2}{3}$ | $\frac{3}{4} \times 1\frac{2}{3}$ | $2\frac{1}{12} \times 1\frac{3}{5}$ | $6\frac{1}{8} \times 1\frac{1}{7}$ |
| 4. $\frac{3}{4} \div \frac{1}{2}$ | $12 \div \frac{2}{5}$ | $4\frac{3}{4} \div \frac{1}{3}$ | $\frac{4}{5} \div 1\frac{2}{3}$ | $\frac{5}{8} \div 3\frac{3}{4}$ |
-
- | | | |
|--------------------------------|-------------------------------|--------------------------------|
| a | b | c |
| 5. $\frac{2}{3} = \frac{6}{?}$ | $\frac{12}{15} = \frac{4}{?}$ | $\frac{45}{50} = \frac{?}{10}$ |

If you made errors in	Row 1	Row 2	Row 3	Row 4	Row 5
Study text pages	38-39	38-39	41	42-43	34, 37
Work Extra Practice Sets	31, 34, 36, 37	35, 38-39	45	50	26, 27

How Well Can You Figure?

Computation Test 2

1. Find the ratio of the first number to the second:

a. $\frac{2}{3} \div \frac{3}{4}$; b. $\frac{2}{3} \div \frac{1}{6}$; c. $25 \div 125$.

2. $\frac{3}{5} + \frac{1}{3} + \frac{1}{4}$ 3. $\frac{5}{6} + 2\frac{1}{2}$ 4. $3\frac{3}{4} + 12\frac{7}{8}$ 5. $6\frac{11}{12} + 5\frac{2}{3}$
 6. $\frac{5}{6} - \frac{2}{3}$ 7. $\frac{1}{4} - \frac{1}{8}$ 8. $3\frac{3}{4} - 1\frac{5}{6}$ 9. $\frac{3}{5} \times \frac{7}{8}$
 10. $\frac{1}{4} \div 16$ 11. $\frac{7}{12} \times 16$ 12. $\frac{3}{4} \div \frac{6}{8}$ 13. $1\frac{9}{10} - 1\frac{1}{2}$
 14. $15 \div \frac{1}{3}$ 15. $\frac{4}{8} \div \frac{1}{4}$ 16. $2\frac{1}{2} \times 8$ 17. $5\frac{1}{2} \times 2\frac{2}{3}$

18. Measure each of the lines A and B, below, to the nearest $\frac{1}{8}$ inch. Then, by dividing, find the ratio of line A to line B.

A _____

B _____

Can You Solve Problems?

Problem Test 2

1. There are 18 boys in the 7th grade and 30 boys in the 8th grade. What is the ratio of the number of boys in the 7th grade to the number of boys in the 8th grade?

2. Mr. Black is $\frac{3}{4}$ as old as Mr. Henry. Mr. Henry is 48 yr. old. How old is Mr. Black?

3. Dick's home is 8 blocks from the school. The post office is 6 blocks farther away. What is the ratio of the distance from Dick's home to school to the distance from his home to the post office? Work carefully!

4. Mary divided a 6-inch line into $\frac{3}{8}$ -inch parts. How many parts were there?

5. Draw the 6-inch line for Ex. 4, marking the parts on it.

6. Draw a line which is $\frac{3}{4}$ as long as a $\frac{1}{2}$ -inch line.

7. Draw a line that is $\frac{2}{3}$ the length of a $2\frac{1}{4}$ -inch line.

8. A candy maker divided $12\frac{1}{2}$ lb. of candy into $\frac{4}{5}$ -ounce pieces. How many pieces did he make?

9. Mrs. Browning bought a $3\frac{1}{2}$ -pound roast, a quarter of a pound of butter, $\frac{1}{2}$ lb. of cheese, and a bunch of grapes that weighed $1\frac{1}{3}$ lb. What was the total weight of her purchases?

10. Find the total cost of 18 lb. of butter at $67\frac{1}{2}\text{¢}$ per pound, $1\frac{2}{3}$ dozen eggs at 63¢ per dozen, and a $3\frac{1}{2}$ -pound steak at 84¢ per pound.

11. The movie house has 840 seats on the main floor, $\frac{3}{5}$ as many in the first balcony as on the main floor, and $\frac{2}{3}$ as many in the second balcony as in the first balcony. What is the total seating capacity of the hall?

12. In our playhouse there are 350 seats on the main floor, 175 in the first balcony and 98 in the second balcony. If the prices per seat are respectively \$2.75, \$1.76, and \$0.88, what would be the total receipts if all seats were sold?

GINN AND COMPANY

10
14
V
7
Δ
90
C
15
1
0
12
8